

Finding the optimal route in a road traffic network

GRAPH THEORY

Networks goes to school
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Nikki Levering
Nicos Starreveld
Mehmet Akif Yildiz



**NET
WORKS**
THENETWORKCENTER.NL



What are networks?

network

noun [C]

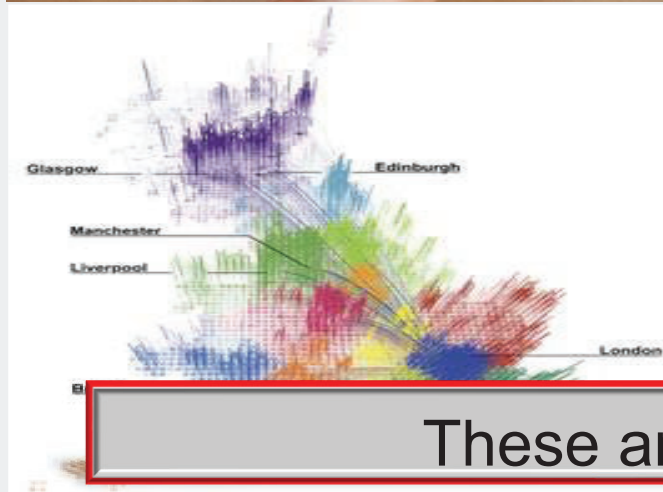
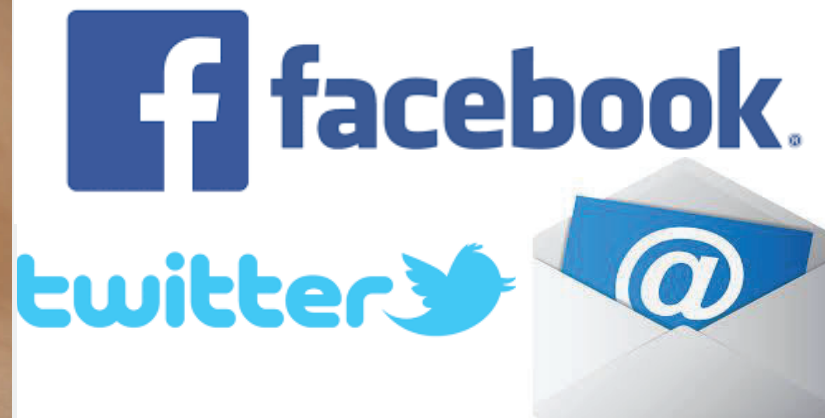
UK /'net.wɜ:k/ US /'net.wɜ:k/



B2

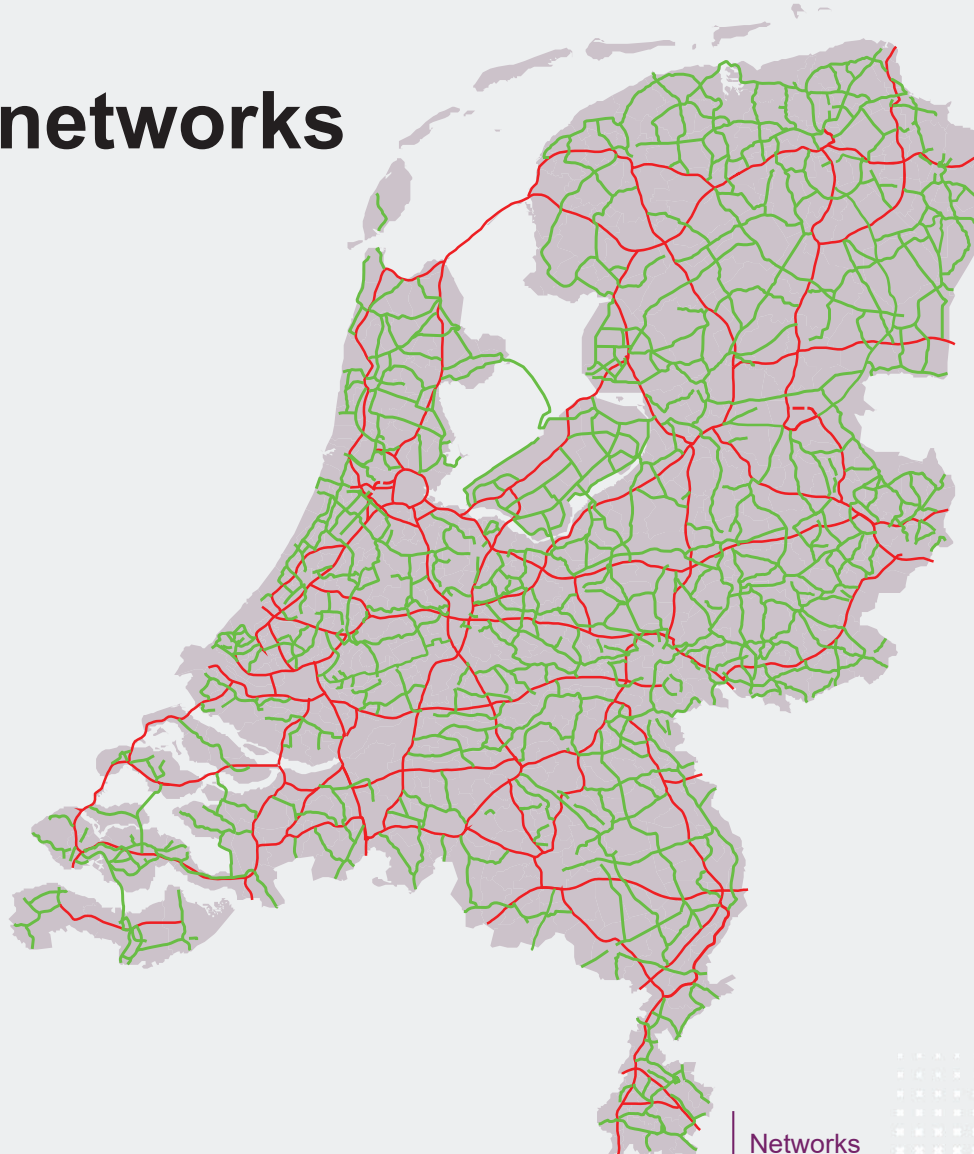
a large system consisting of many similar parts that are connected together to allow movement or communication between or along the parts

What are networks?



These are all networks!

Road networks



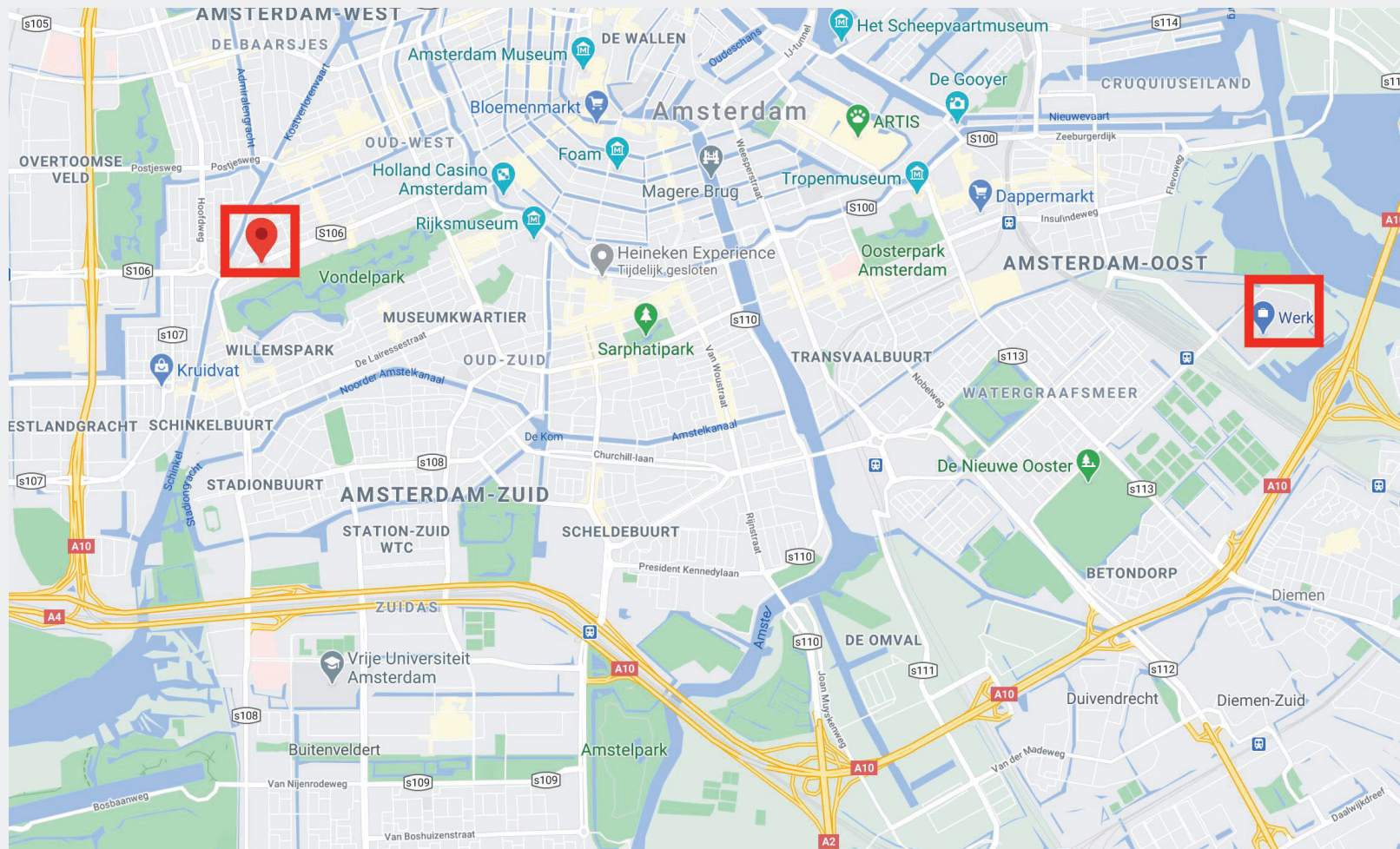
Networks

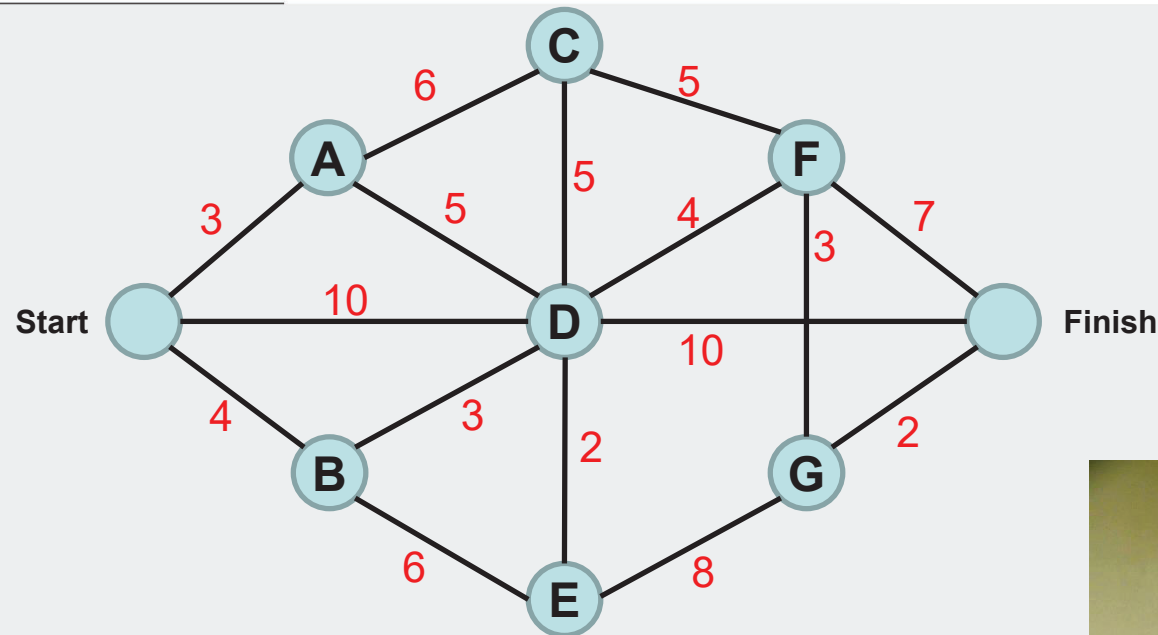
Why study networks?

- What is the shortest route to reach my destination?
- What route is least likely to suffer from traffic jams?
- What route should I take to arrive at my destination *in time*?



Finding the shortest path can be difficult

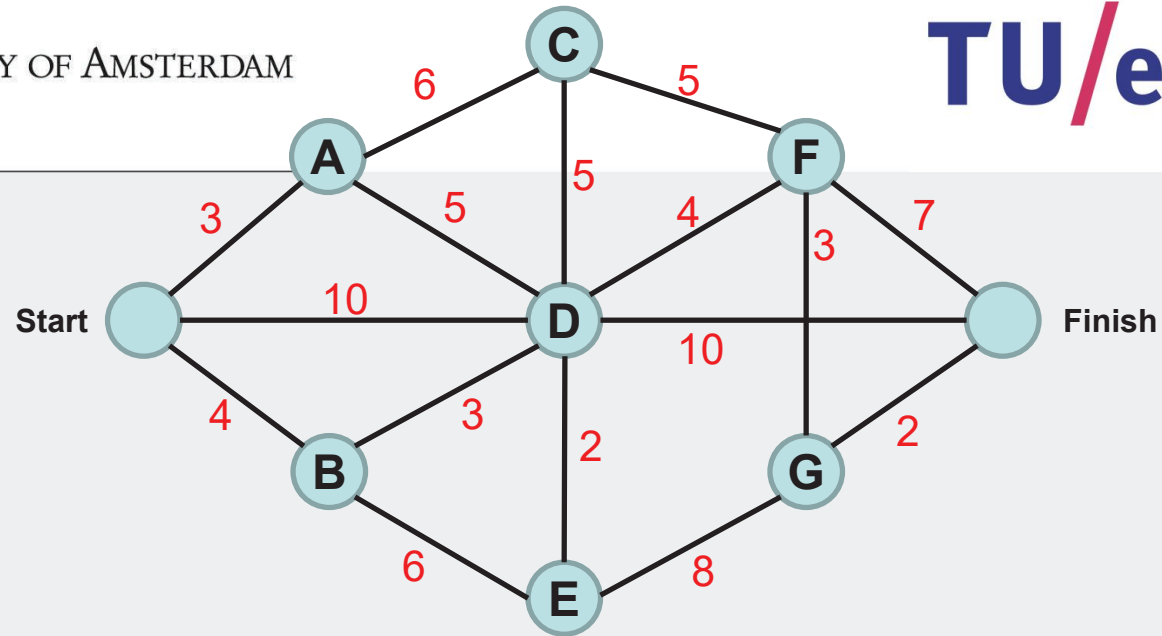




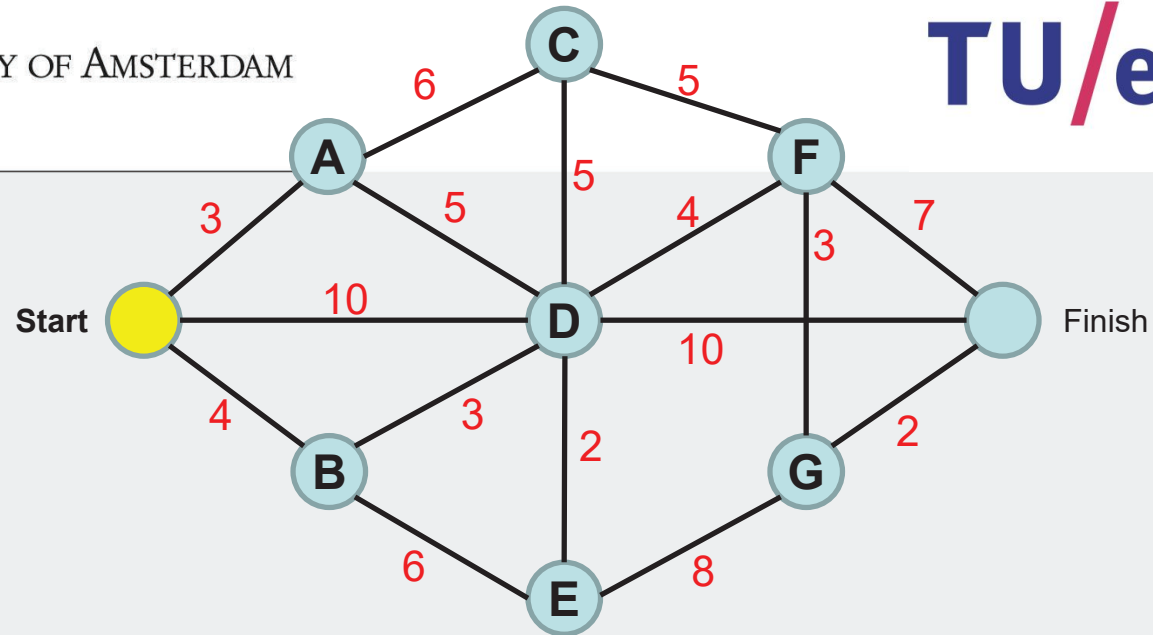
- What is the shortest route from start to finish?
- Dijkstra's algorithm can be used to determine the shortest route in a network



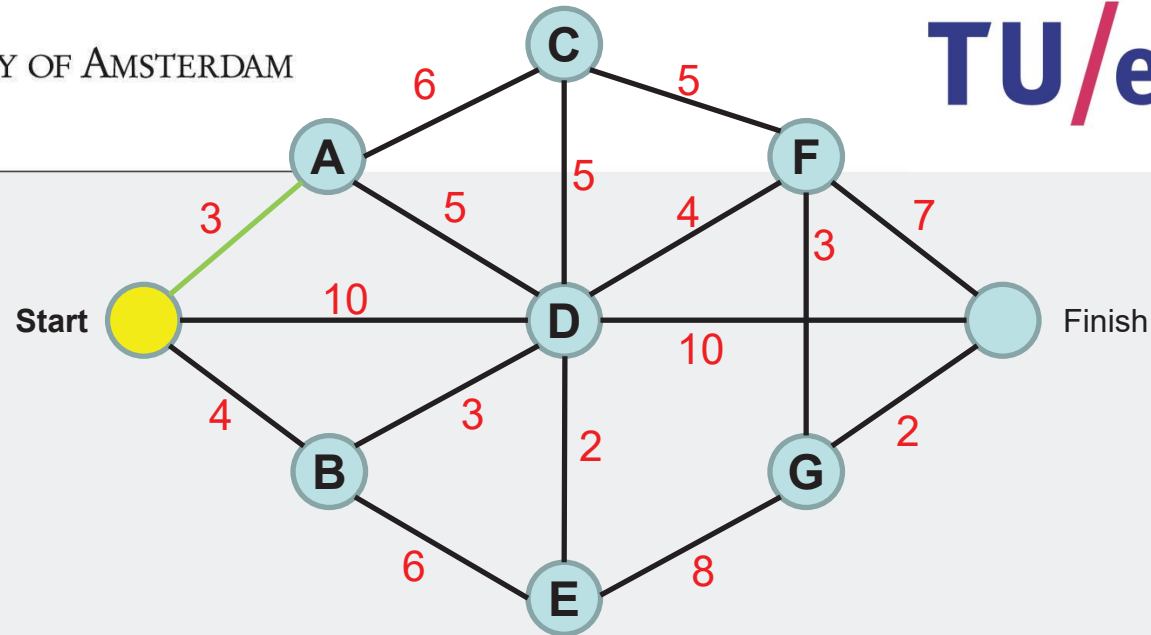
Edsger W. Dijkstra
(1930-2002)



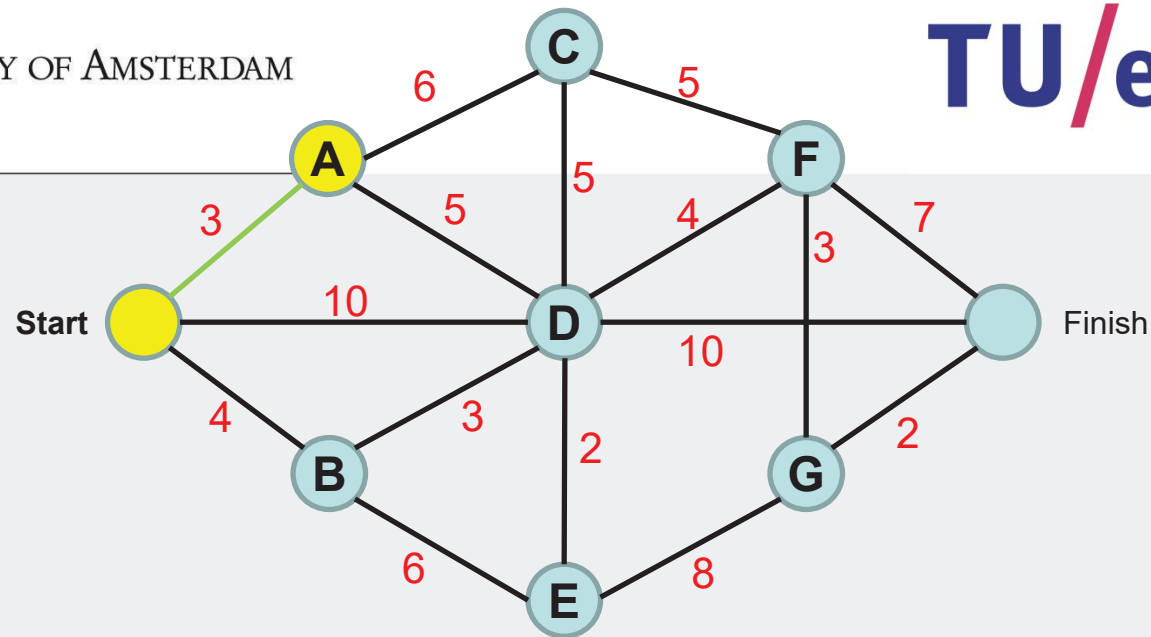
A	B	C	D	E	F	G	Finish



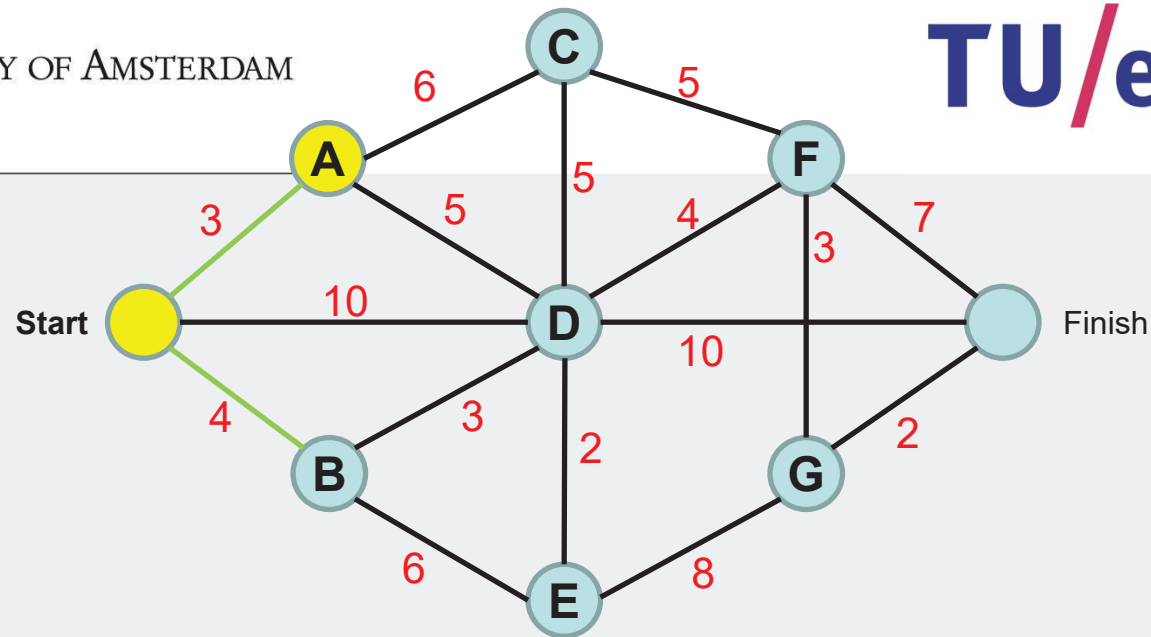
A	B	C	D	E	F	G	Finish
3, start	4, start		10, start				



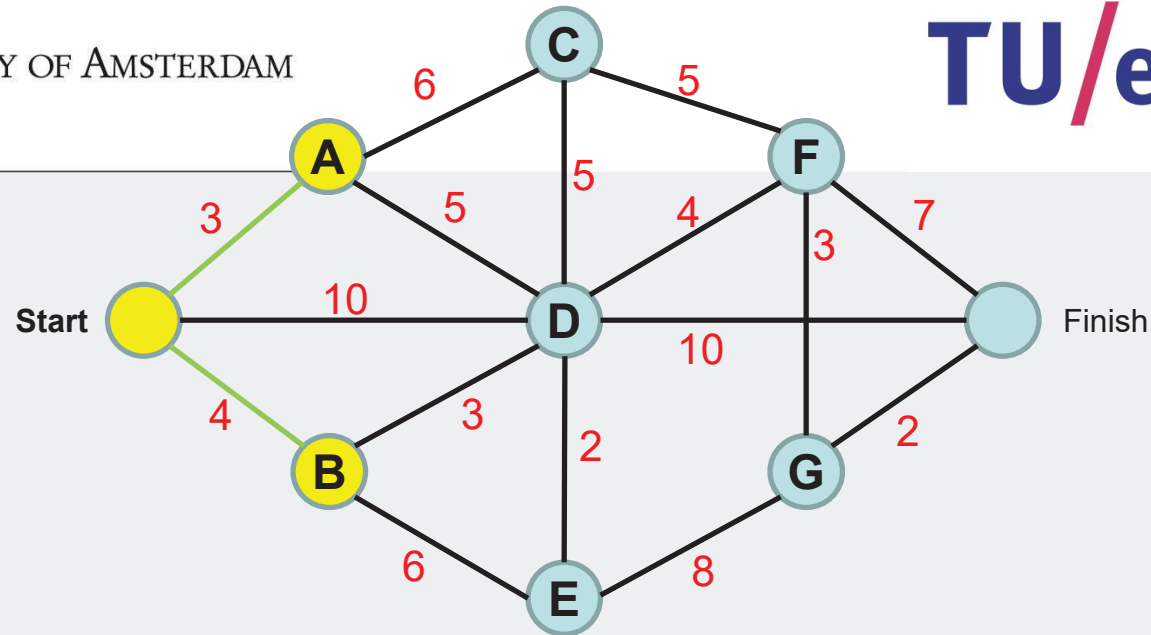
A	B	C	D	E	F	G	Finish
3, start	4, start		10, start				



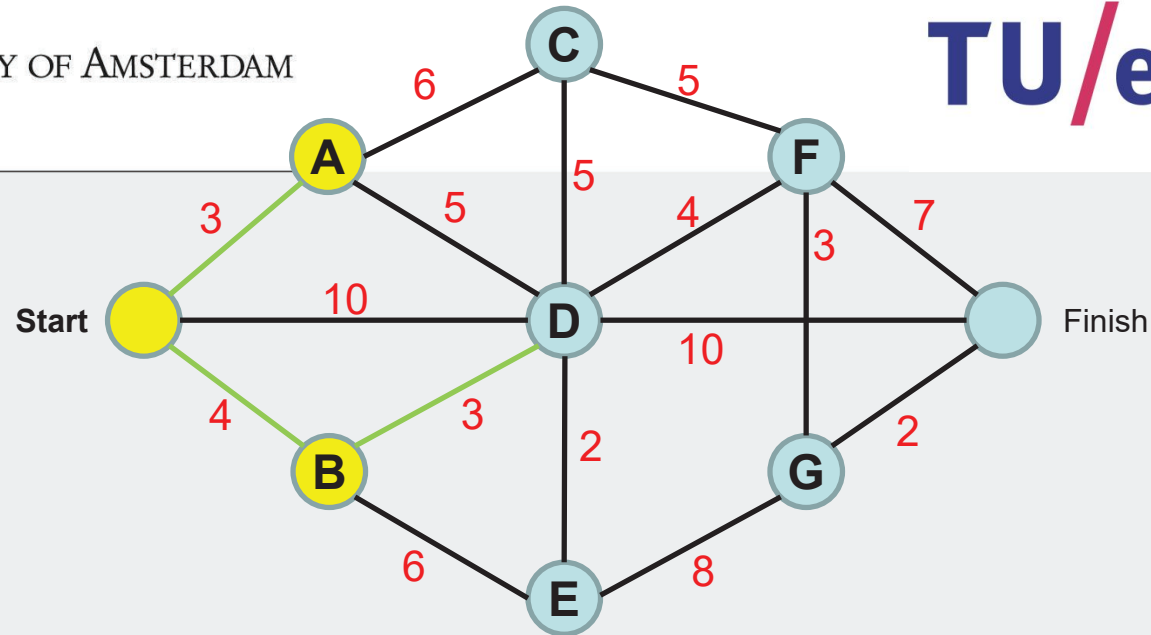
A	B	C	D	E	F	G	Finish
3, start	4, start		10, start				
*	4, start	9, A	8, A				
*							
*							
*							
*							
*							
*							



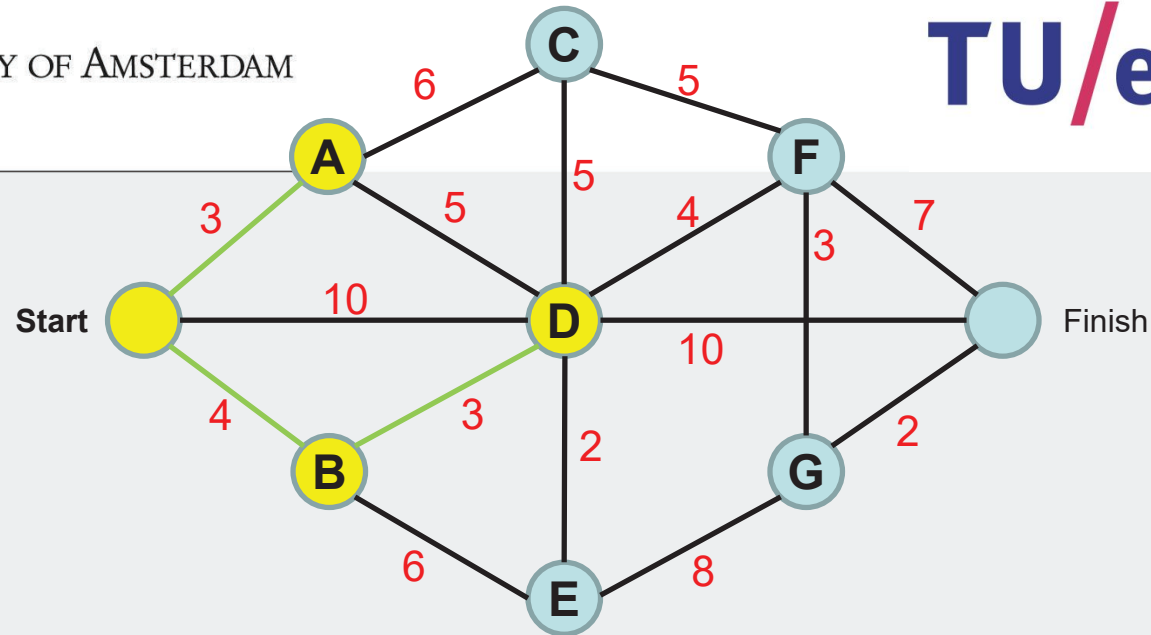
A	B	C	D	E	F	G	Finish
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*	4, start	9, A	8, A				
*							
*							
*							
*							
*							
*							



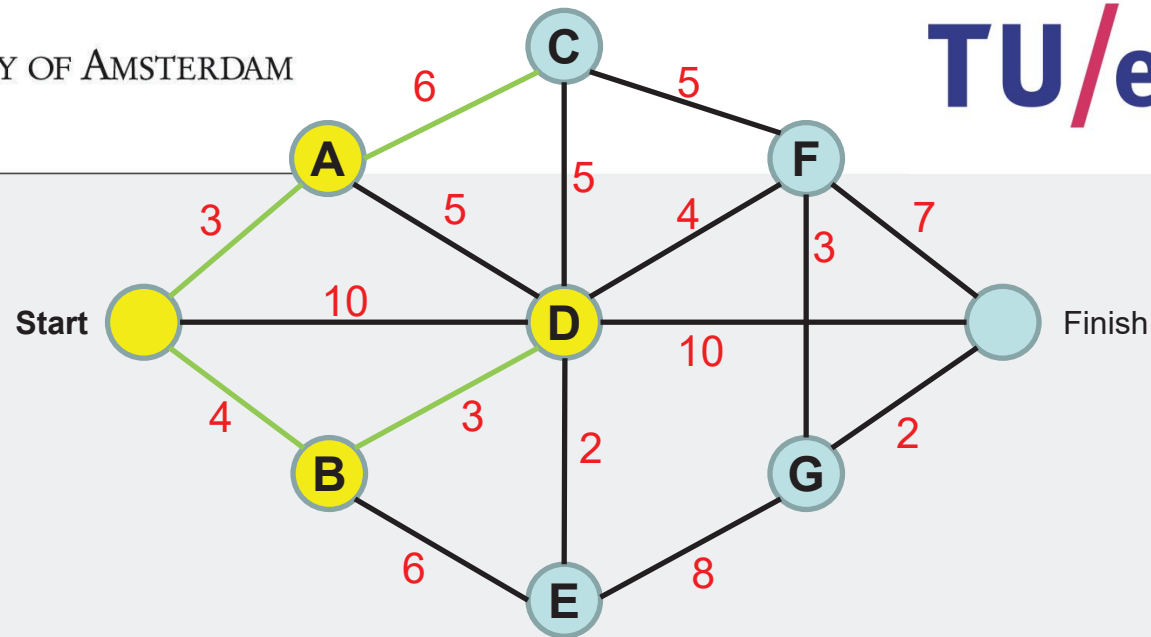
A	B	C	D	E	F	G	Finish
3, start	4, start		10, start				
*	4, start	9, A	8, A				
*	*	9, A	7, B	10, B			
*	*						
*	*						
*	*						
*	*						
*	*						



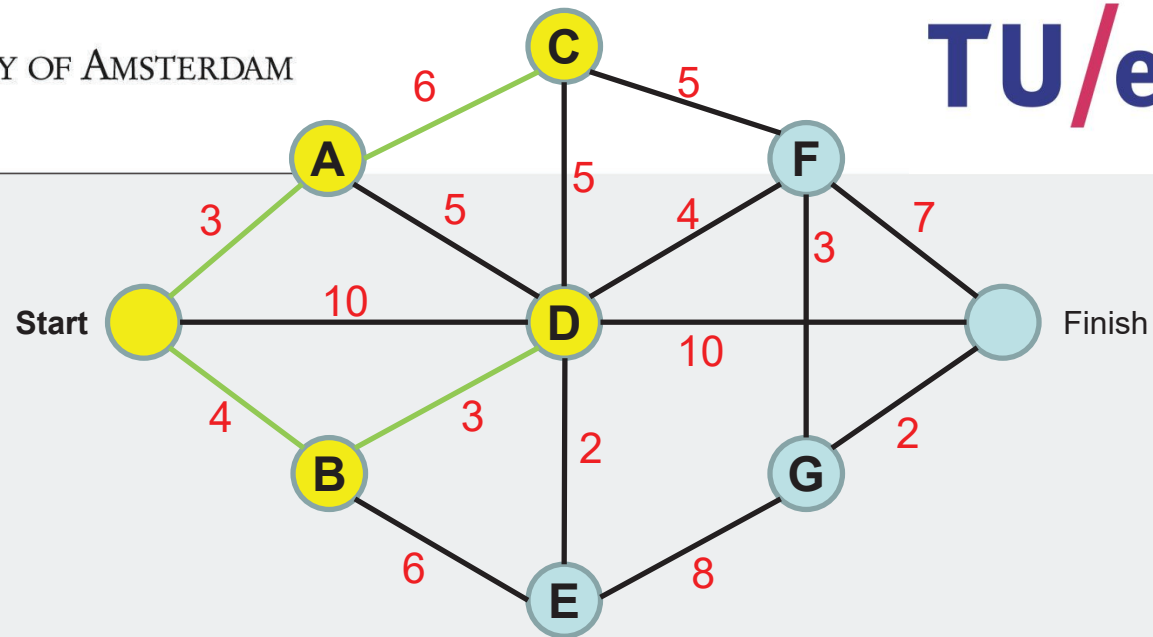
A	B	C	D	E	F	G	Finish
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*	4, start	9, A	8, A				
*	*	9, A	7, B	10, B			
*	*						
*	*						
*	*						
*	*						
*	*						



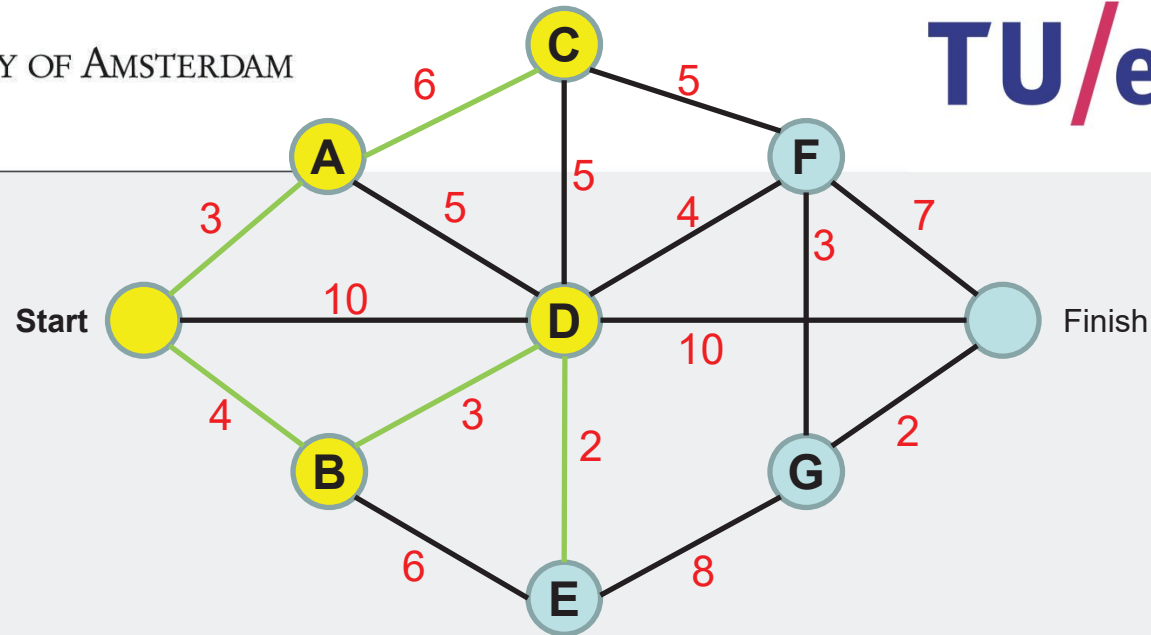
A	B	C	D	E	F	G	Finish
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*	4, start	9, A	8, A				
*	*	9, A	7, B	10, B			
*	*	9, A	*	9, D	11, D		17, D
*	*		*				
*	*		*				
*	*		*				
*	*		*				



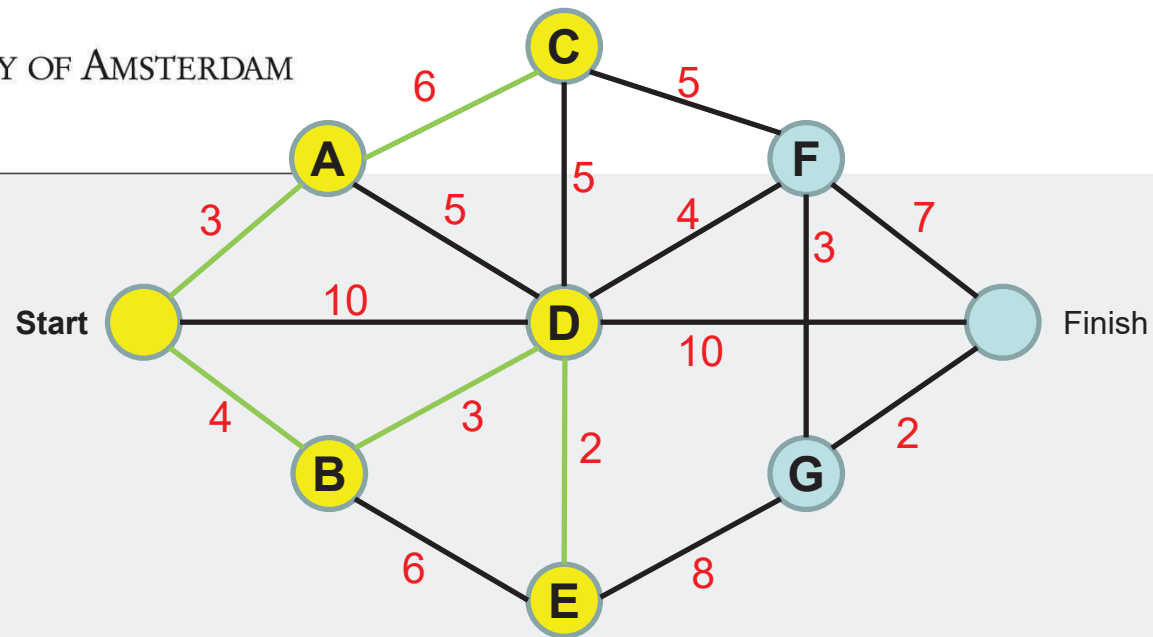
A	B	C	D	E	F	G	Finish
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*	4, start	9, A	8, A				
*	*	9, A	7, B	10, B			
*	*	9, A	*	9, D	11, D		17, D
*	*		*				
*	*		*				
*	*		*				
*	*		*				



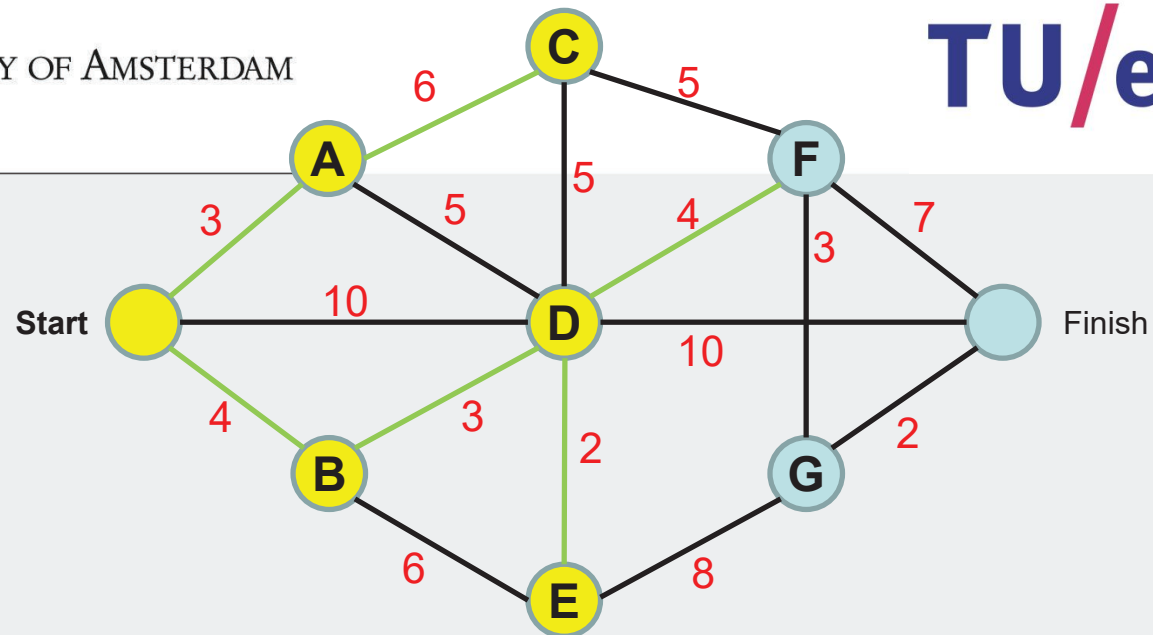
A	B	C	D	E	F	G	Finish
3, start	4, start		10, start				
*	4, start	9, A	8, A				
*	*	9, A	7, B	10, B			
*	*	9, A	*	9, D	11, D		17, D
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*	*	*	*				



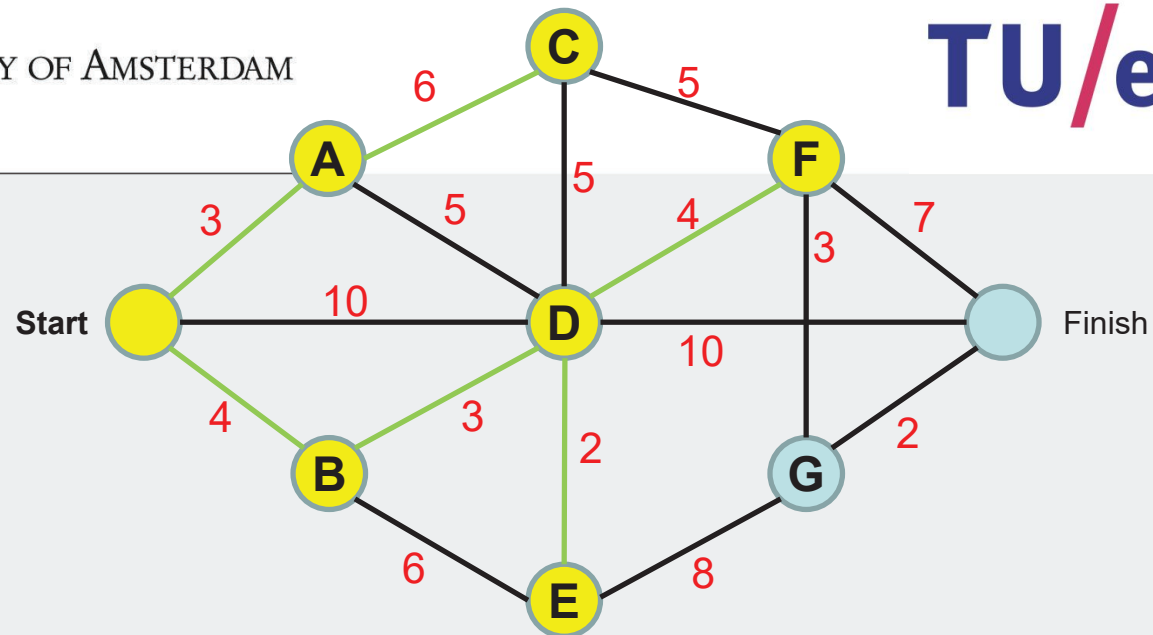
A	B	C	D	E	F	G	Finish
3, start	4, start		10, start				
*	4, start	9, A	8, A				
*	*	9, A	7, B	10, B			
*	*	9, A	*	9, D	11, D		17, D
*	*	*	*	9, D	11, D		17, D
*	*	*	*				
*	*	*	*				
*	*	*	*				



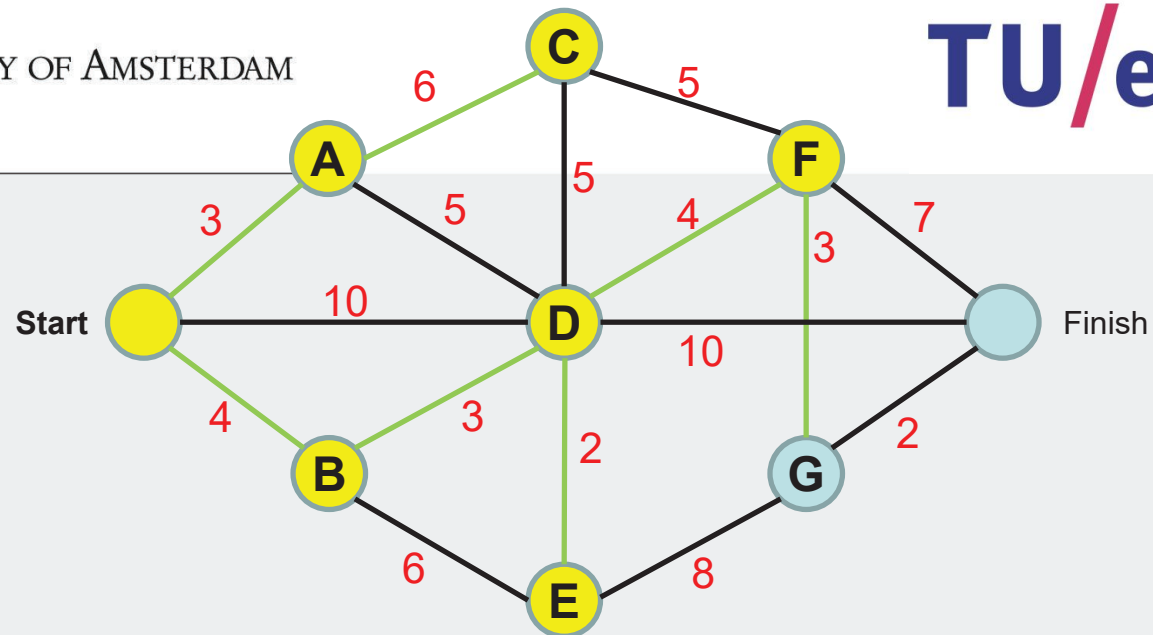
A	B	C	D	E	F	G	Finish
3, start	4, start		10, start				
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*	*	9, A	7, B	10, B			
*	*	9, A	*	9, D	11, D		17, D
*	*	*	*	9, D	11, D		17, D
*	*	*	*	*	11, D	17, E	17, D
*	*	*	*	*			
*	*	*	*	*			



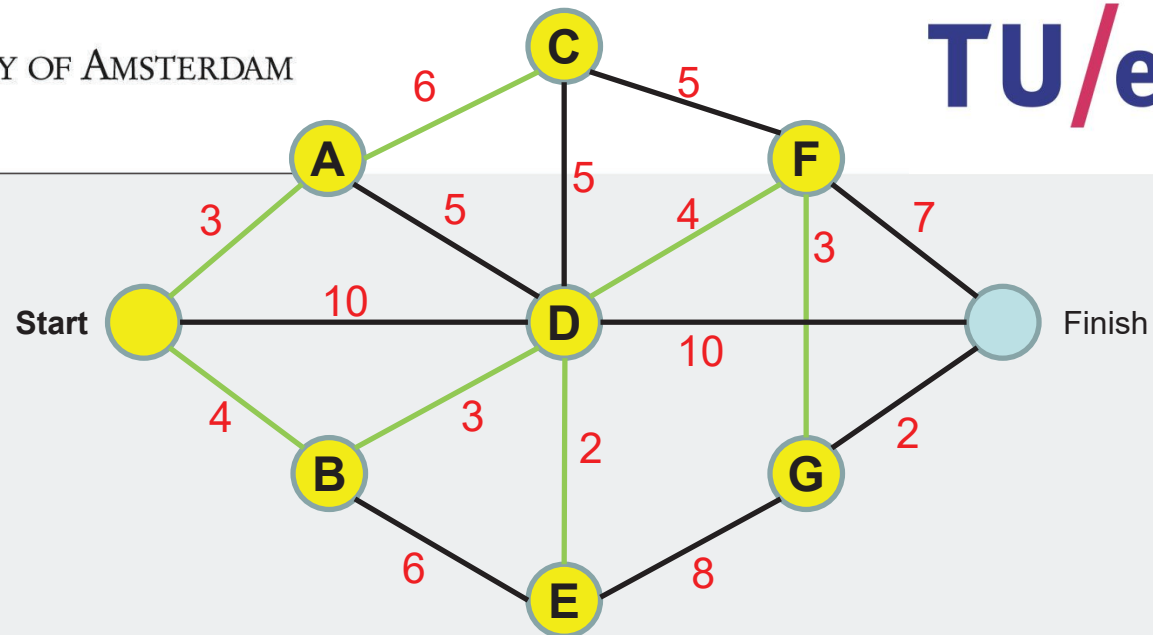
A	B	C	D	E	F	G	Finish
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*	4, start	9, A	8, A				
*	*	9, A	7, B	10, B			
*	*	9, A	*	9, D	11, D		17, D
*	*	*	*	9, D	11, D		17, D
*	*	*	*	*	11, D	17, E	17, D
*	*	*	*	*			
*	*	*	*	*			



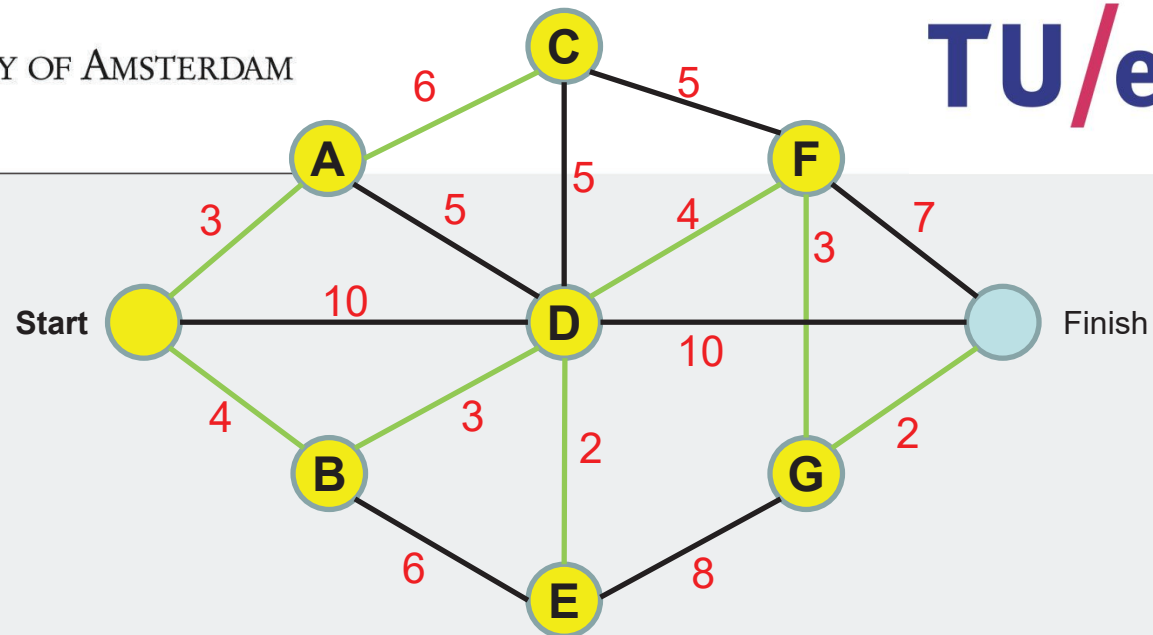
A	B	C	D	E	F	G	Finish
3, start	4, start		10, start				
*	4, start	9, A	8, A				
*	*	9, A	7, B	10, B			
*	*	9, A	*	9, D	11, D		17, D
*	*	*	*	9, D	11, D		17, D
*	*	*	*	*	11, D	17, E	17, D
*	*	*	*	*	*	14, F	17, D
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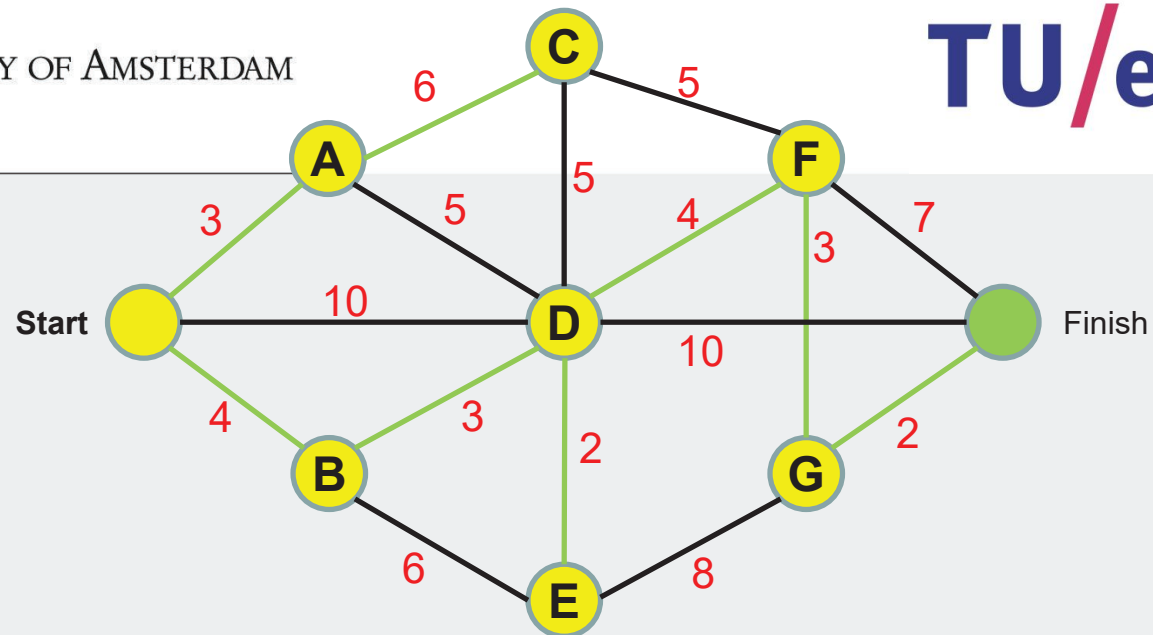
A	B	C	D	E	F	G	Finish
3, start	4, start		10, start				
*	4, start	9, A	8, A				
*	*	9, A	7, B	10, B			
*	*	9, A	*	9, D	11, D		17, D
*	*	*	*	9, D	11, D		17, D
*	*	*	*	*	11, D	17, E	17, D
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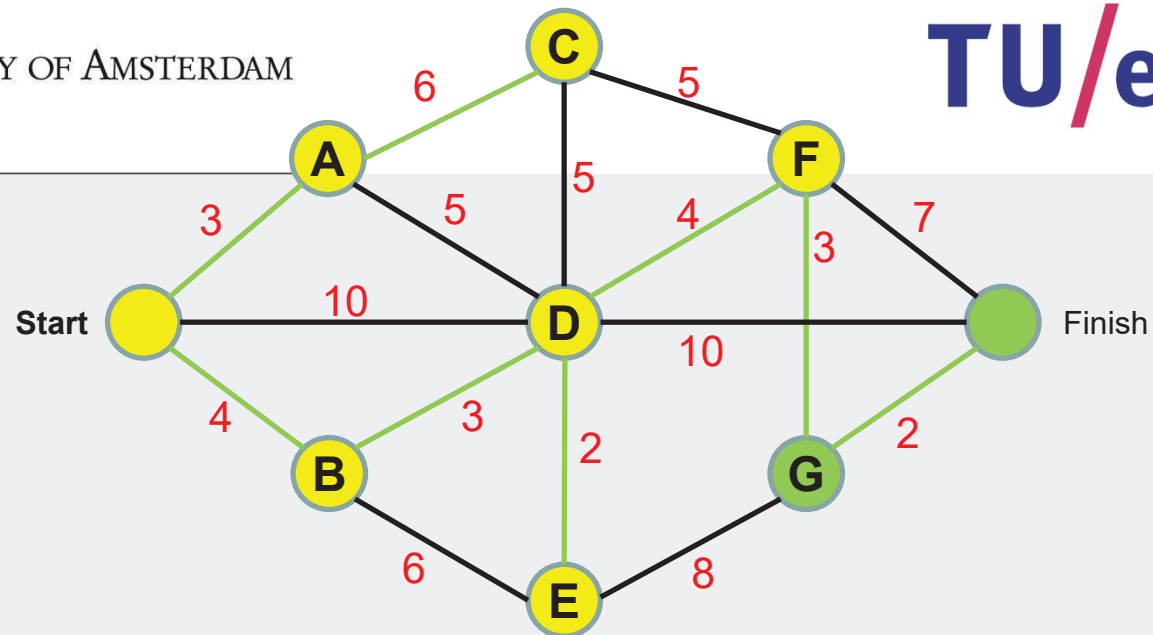
A	B	C	D	E	F	G	Finish
3, start	4, start		10, start				
*	4, start	9, A	8, A				
*	*	9, A	7, B	10, B			
*	*	9, A	*	9, D	11, D		17, D
*	*	*	*	9, D	11, D		17, D
*	*	*	*	*	11, D	17, E	17, D
*	*	*	*	*	*	14, F	17, D
*	*	*	*	*	*	*	16, G



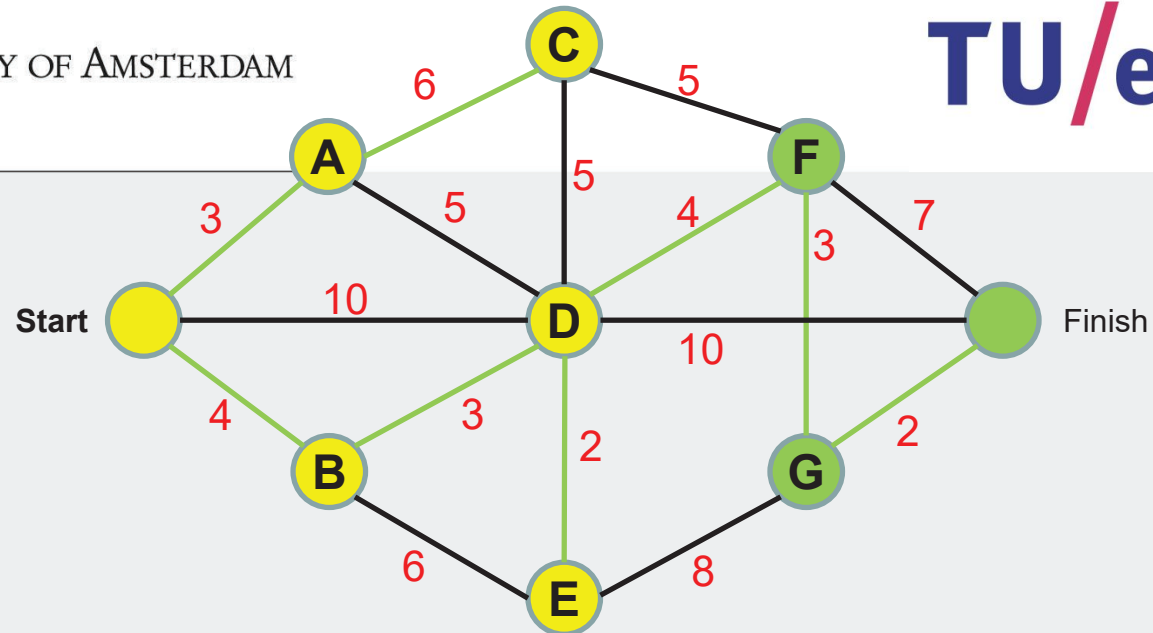
A	B	C	D	E	F	G	Finish
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*	4, start	9, A	8, A				
*	*	9, A	7, B	10, B			
*	*	9, A	*	9, D	11, D		17, D
*	*	*	*	9, D	11, D		17, D
*	*	*	*	*	11, D	17, E	17, D
*	*	*	*	*	*	14, F	17, D
*	*	*	*	*	*	*	16, G



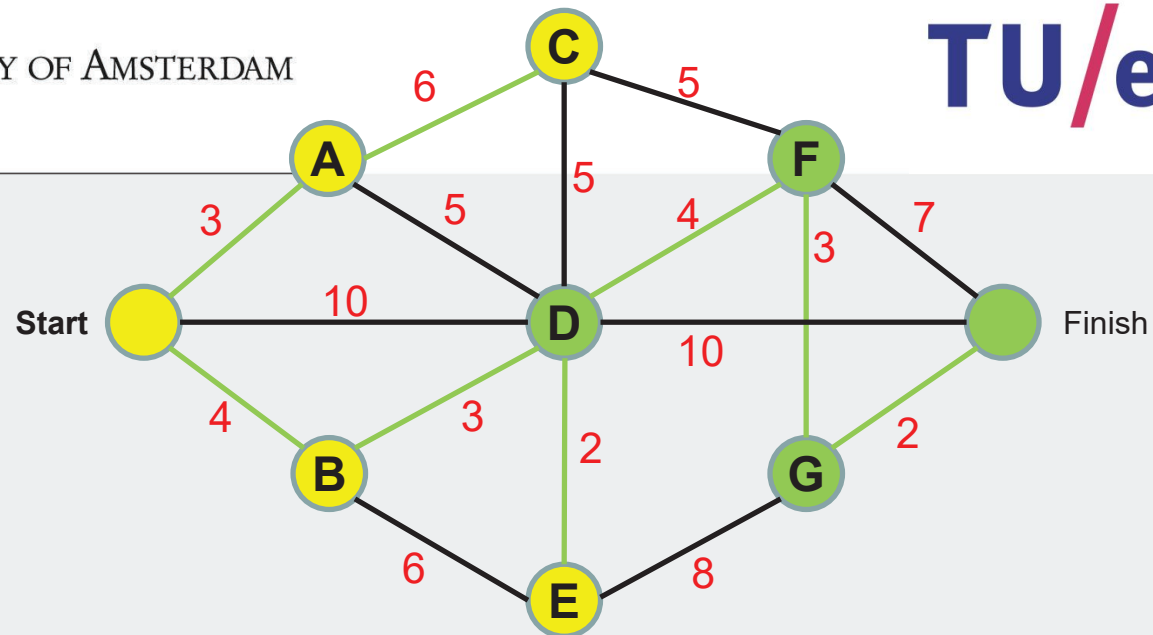
A	B	C	D	E	F	G	Finish
3, start	4, start		10, start				
*	4, start	9, A	8, A				
*	*	9, A	7, B	10, B			
*	*	9, A	*	9, D	11, D		17, D
*	*	*	*	9, D	11, D		17, D
*	*	*	*	*	11, D	17, E	17, D
*	*	*	*	*	*	14, F	17, D
*	*	*	*	*	*	*	16, G



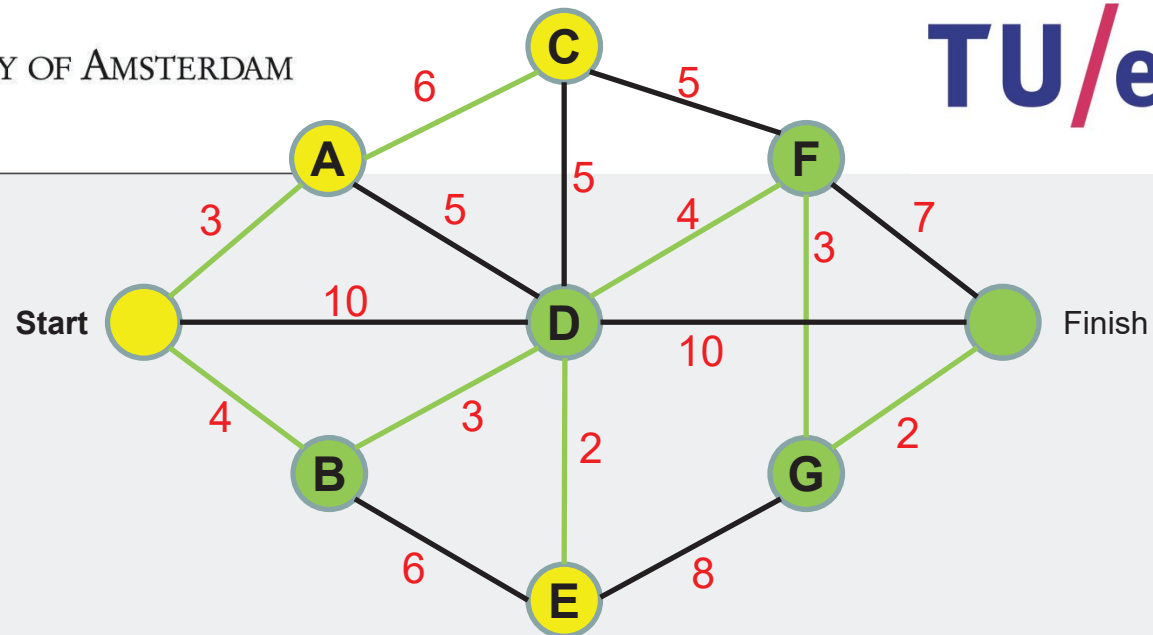
A	B	C	D	E	F	G	Finish
3, start	4, start		10, start				
*	4, start	9, A	8, A				
*	*	9, A	7, B	10, B			
*	*	9, A	*	9, D	11, D		17, D
*	*	*	*	9, D	11, D		17, D
*	*	*	*	*	11, D	17, E	17, D
*	*	*	*	*	*	14, F	17, D
*	*	*	*	*	*	*	16, G



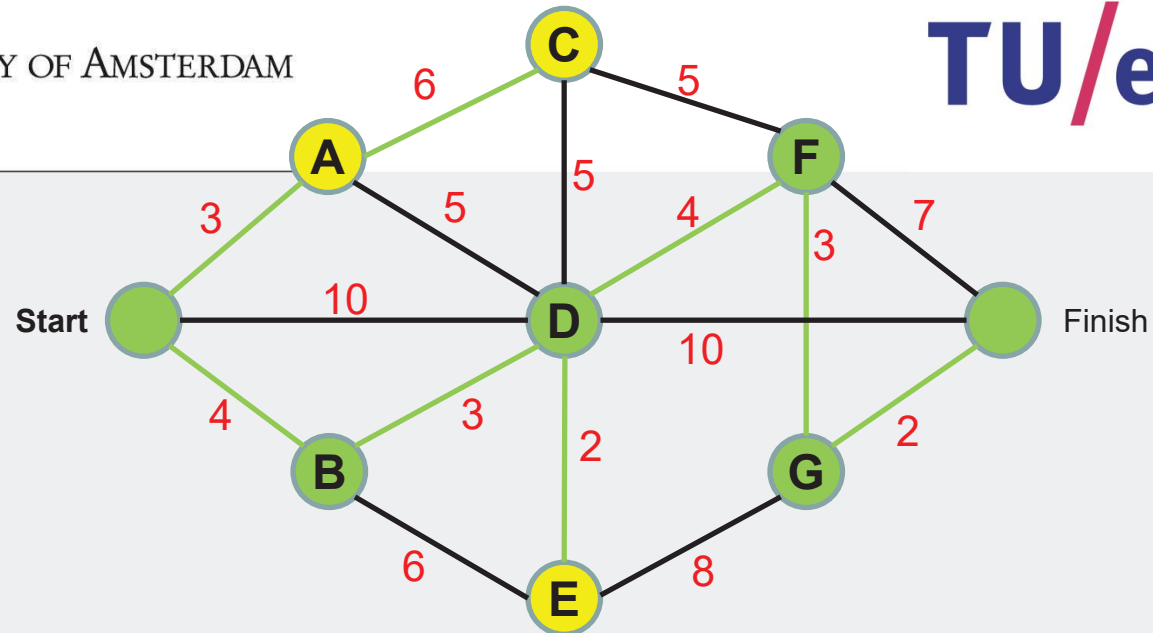
A	B	C	D	E	F	G	Finish
3, start	4, start		10, start				
*	4, start	9, A	8, A				
*	*	9, A	7, B	10, B			
*	*	9, A	*	9, D	11, D		17, D
*	*	*	*	9, D	11, D		17, D
*	*	*	*	*	11, D	17, E	17, D
*	*	*	*	*	*	14, F	17, D
*	*	*	*	*	*	*	16, G



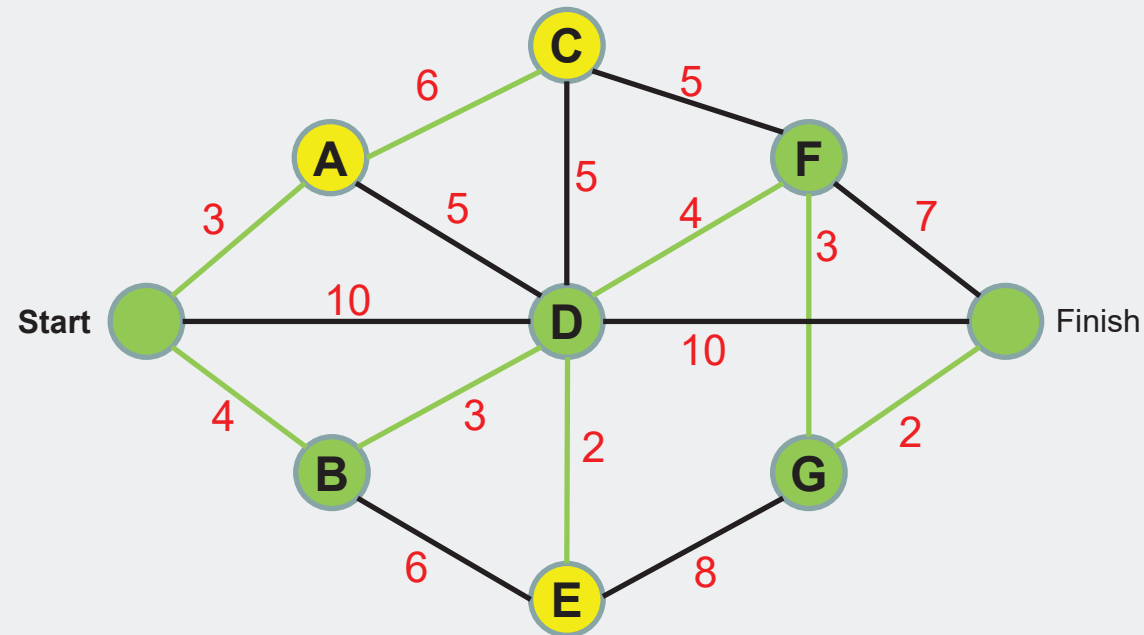
A	B	C	D	E	F	G	Finish
3, start	4, start		10, start				
*	4, start	9, A	8, A				
*	*	9, A	7, B	10, B			
*	*	9, A	*	9, D	11, D		17, D
*	*	*	*	9, D	11, D		17, D
*	*	*	*	*	11, D	17, E	17, D
*	*	*	*	*	*	14, F	17, D
*	*	*	*	*	*	*	16, G



A	B	C	D	E	F	G	Finish
3, start	4, start		10, start				
*	4, start	9, A	8, A				
*	*	9, A	7, B	10, B			
*	*	9, A	*	9, D	11, D		17, D
*	*	*	*	9, D	11, D		17, D
*	*	*	*	*	11, D	17, E	17, D
*	*	*	*	*	*	14, F	17, D
*	*	*	*	*	*	*	16, G



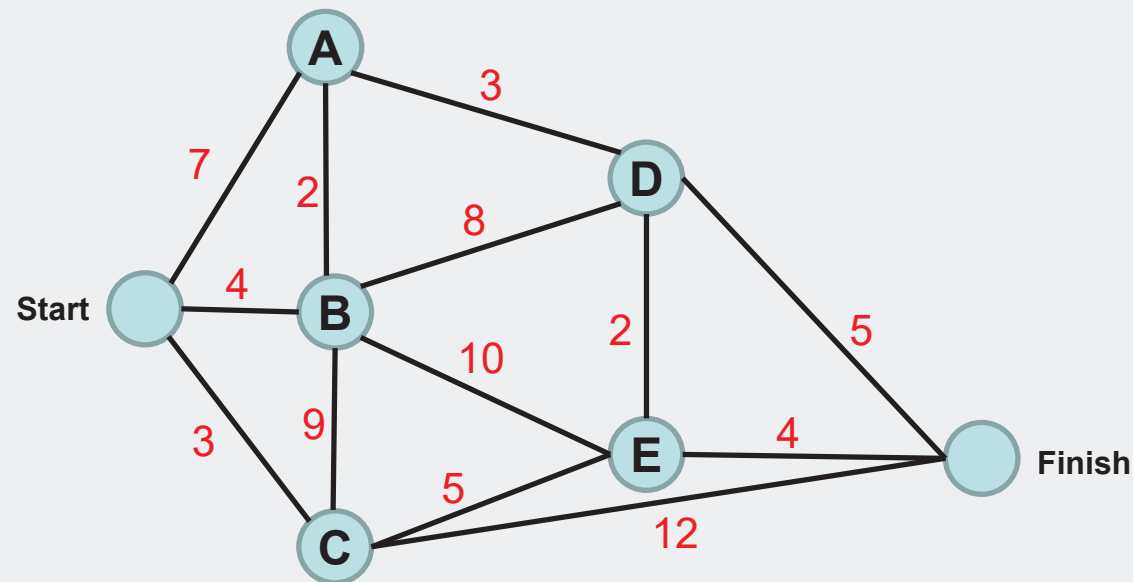
A	B	C	D	E	F	G	Finish
3, start	4, start		10, start				
*	4, start	9, A	8, A				
*	*	9, A	7, B	10, B			
*	*	9, A	*	9, D	11, D		17, D
*	*	*	*	9, D	11, D		17, D
*	*	*	*	*	11, D	17, E	17, D
*	*	*	*	*	*	14, F	17, D
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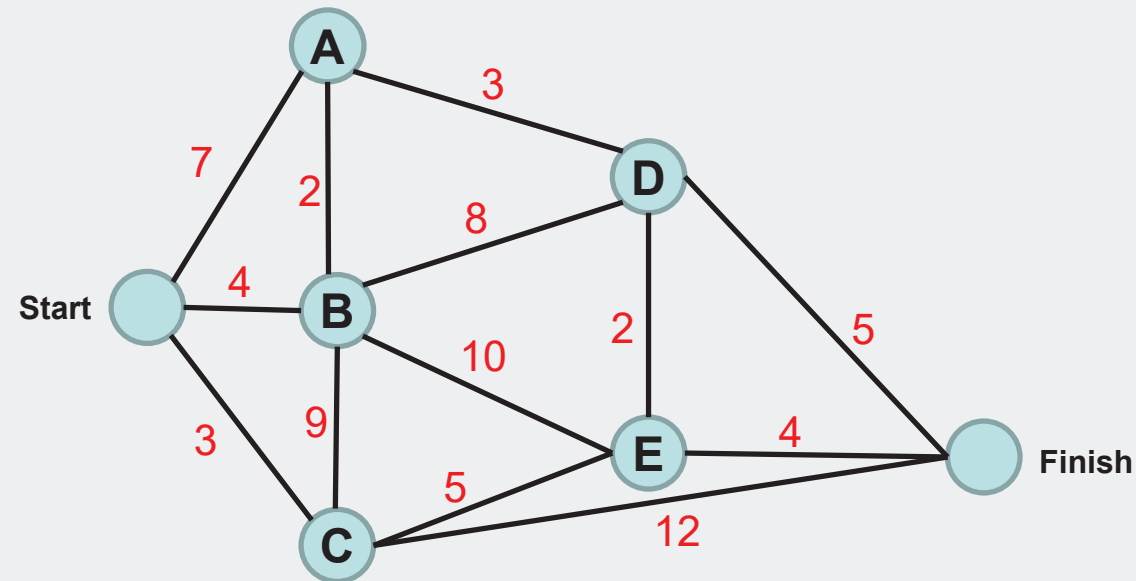


- The shortest route is given by: Start→B→D→F→G→Finish
- We made a mistake! We actually wanted to travel to node E!

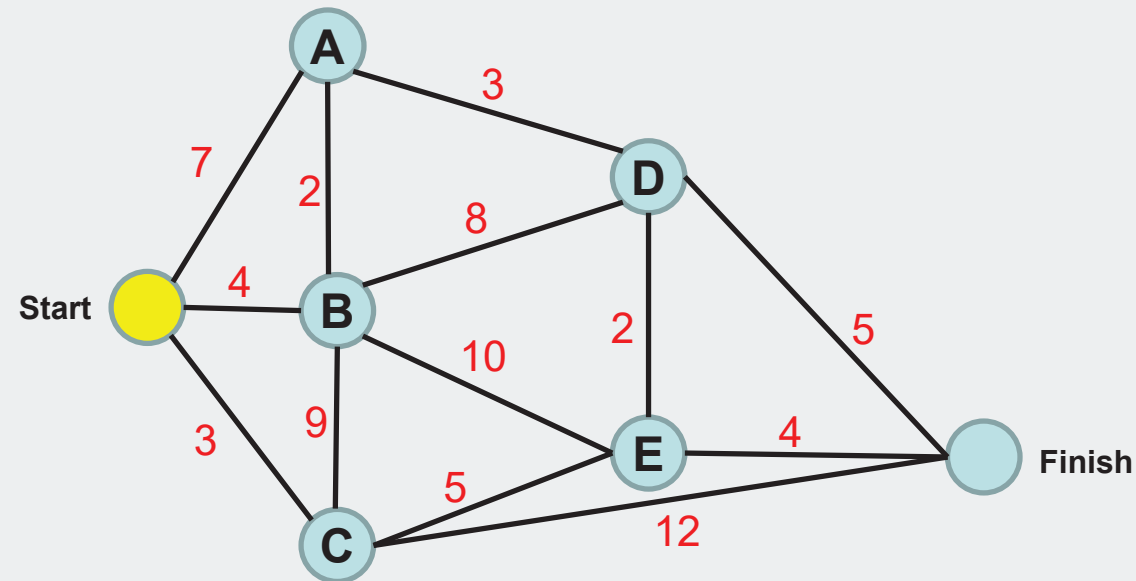
Exercise

- What is the shortest route from start to finish in the following network?

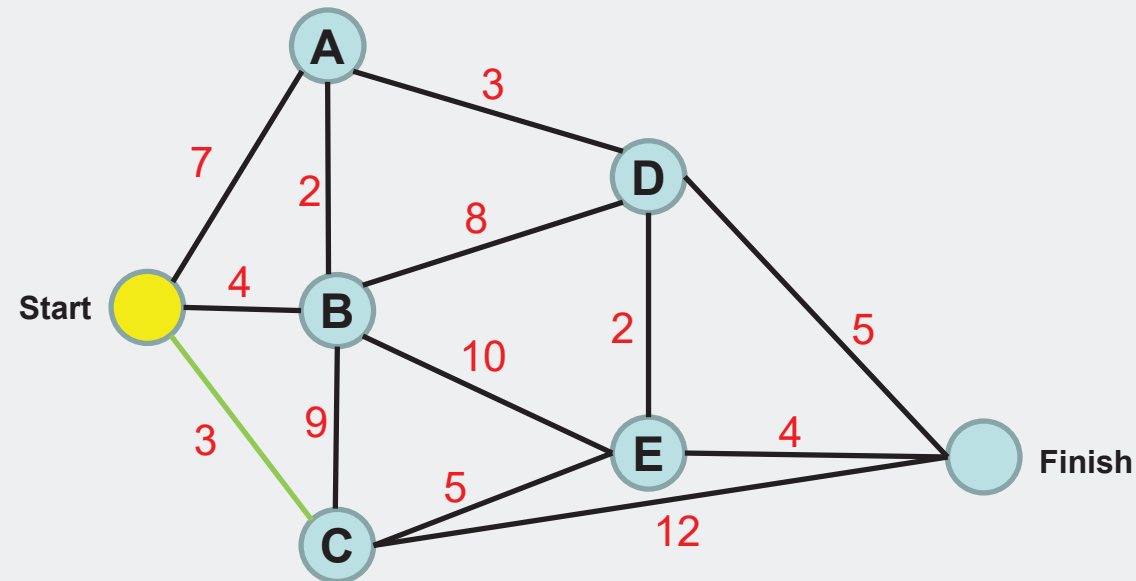




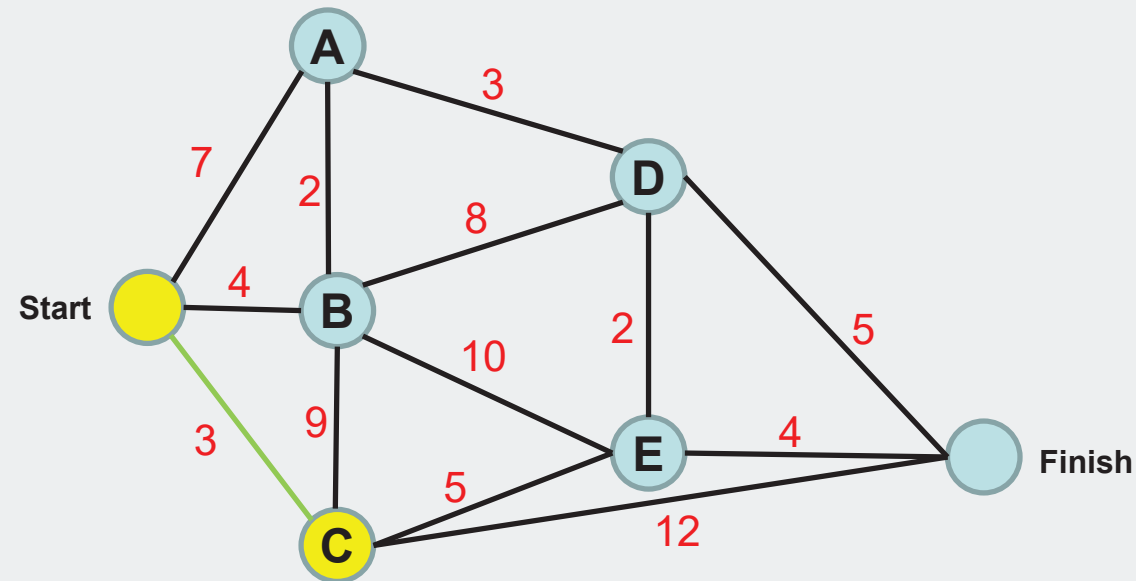
A	B	C	D	E	Finish



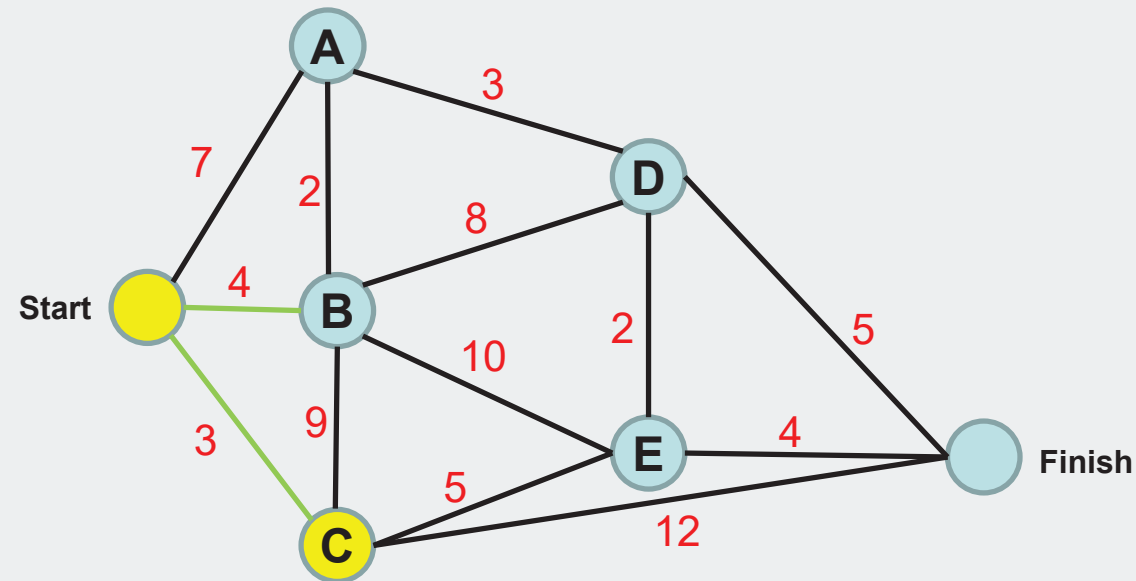
A	B	C	D	E	Finish
7, start	4, start	3, start			



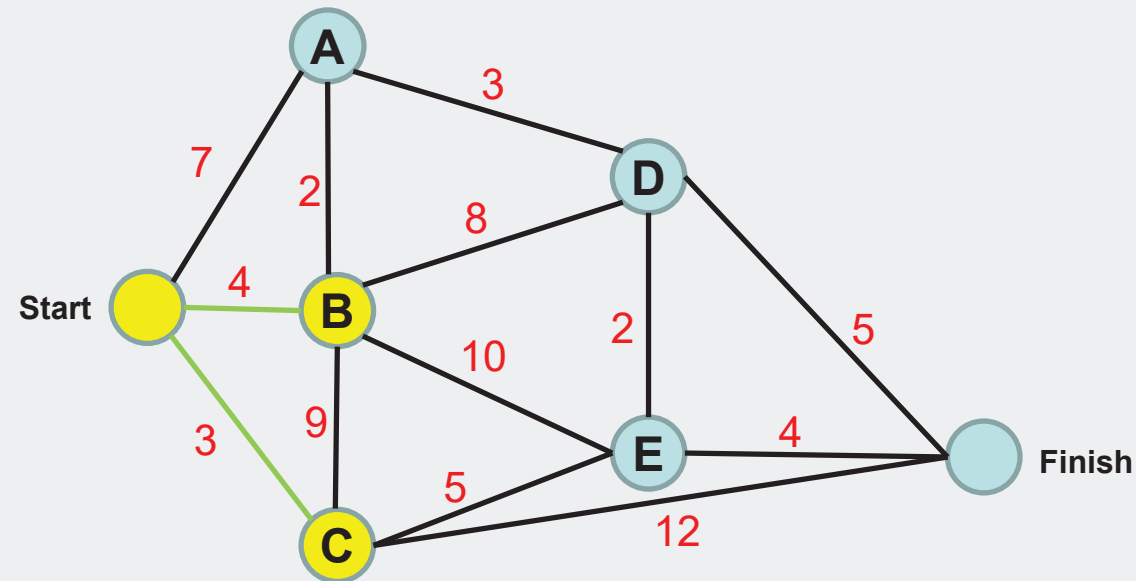
A	B	C	D	E	Finish
7, start	4, start	3, start			



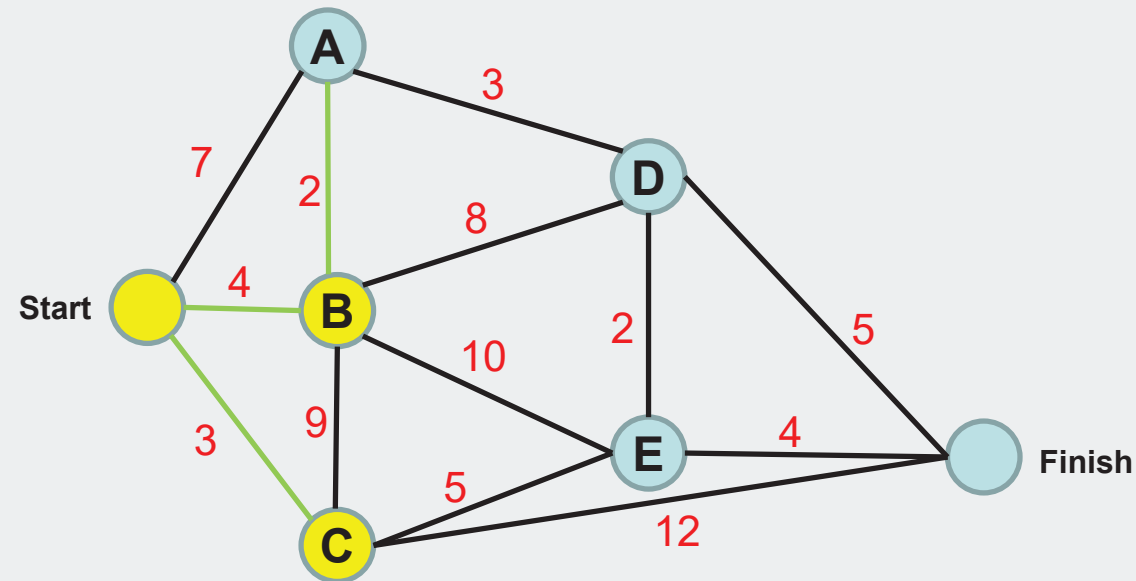
A	B	C	D	E	Finish
7, start	4, start	3, start			
7, start	4, start	*		8, C	15, C
		*			
		*			
		*			
		*			



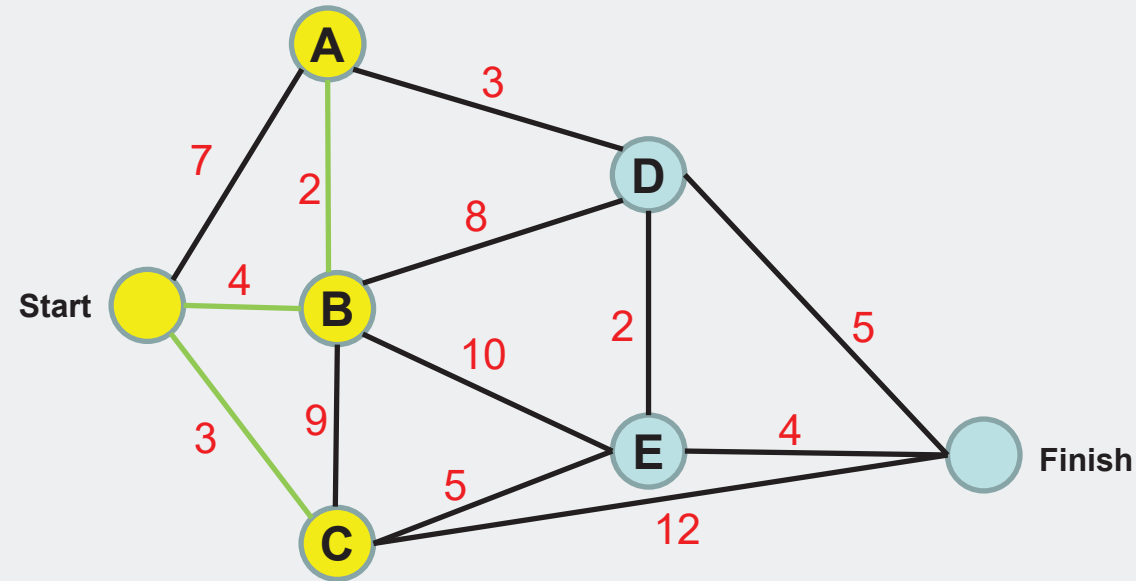
A	B	C	D	E	Finish
7, start	4, start	3, start			
7, start	4, start	*		8, C	15, C
		*			
		*			
		*			
		*			



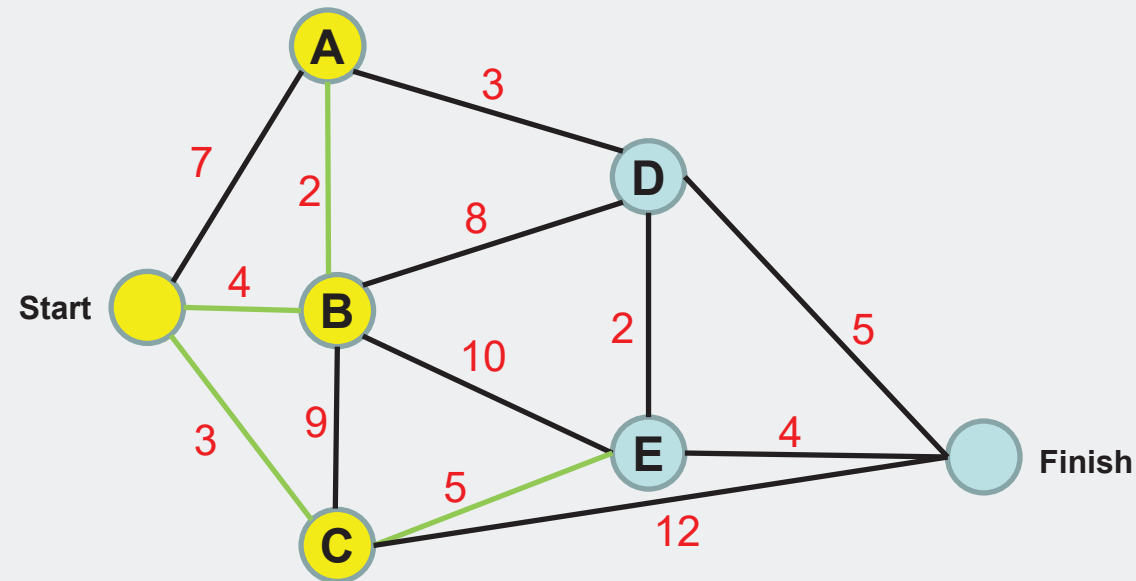
A	B	C	D	E	Finish
7, start	4, start	3, start			
7, start	4, start	*		8, C	15, C
6, B	*	*	12, B	8, C	15, C
	*	*			
	*	*			
	*	*			



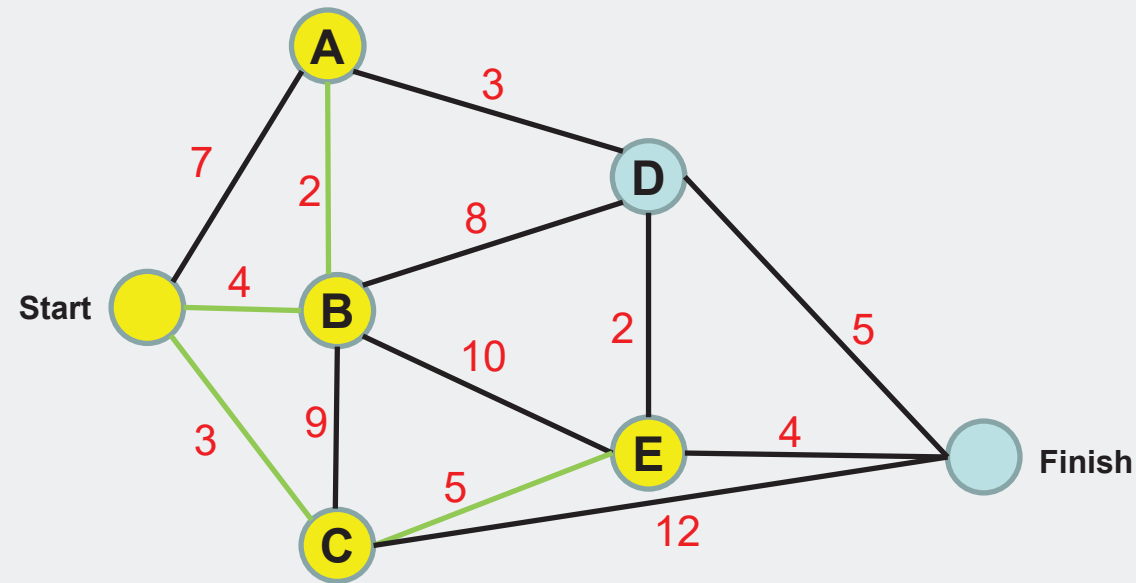
A	B	C	D	E	Finish
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7, start	4, start	*		8, C	15, C
6, B	*	*	12, B	8, C	15, C
	*	*			
	*	*			
	*	*			



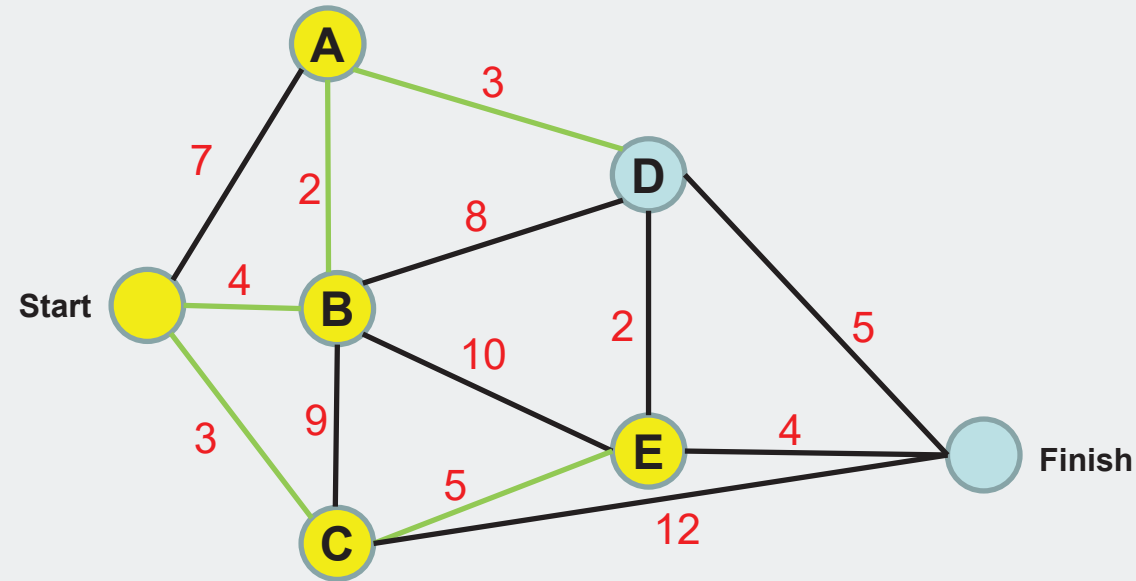
A	B	C	D	E	Finish
7, start	4, start	3, start			
7, start	4, start	*		8, C	15, C
6, B	*	*	12, B	8, C	15, C
*	*	*	9, A	8, C	15, C
*	*	*			
*	*	*			



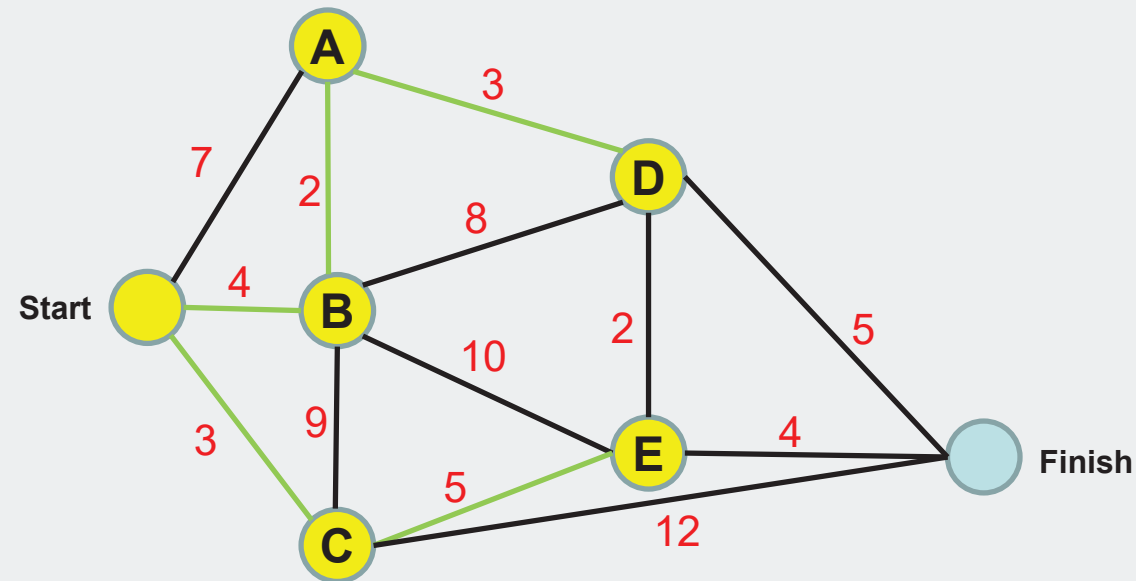
A	B	C	D	E	Finish
7, start	4, start	3, start			
7, start	4, start	*		8, C	15, C
6, B	*	*	12, B	8, C	15, C
*	*	*	9, A	8, C	15, C
*	*	*			
*	*	*			



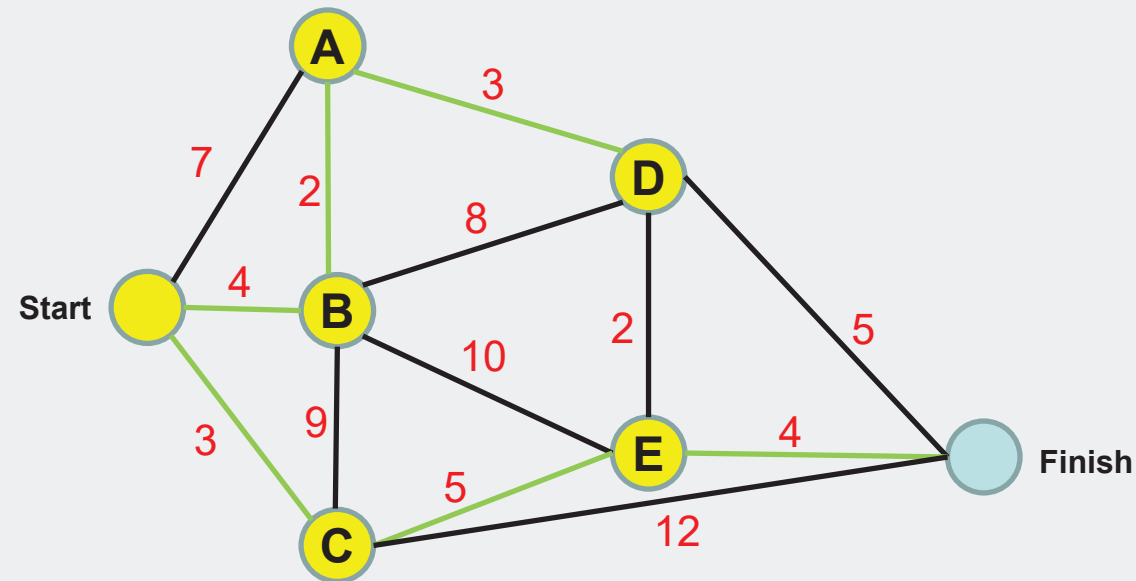
A	B	C	D	E	Finish
7, start	4, start	3, start			
7, start	4, start	*		8, C	15, C
6, B	*	*	12, B	8, C	15, C
*	*	*	9, A	8, C	15, C
*	*	*	9, A	*	12, E
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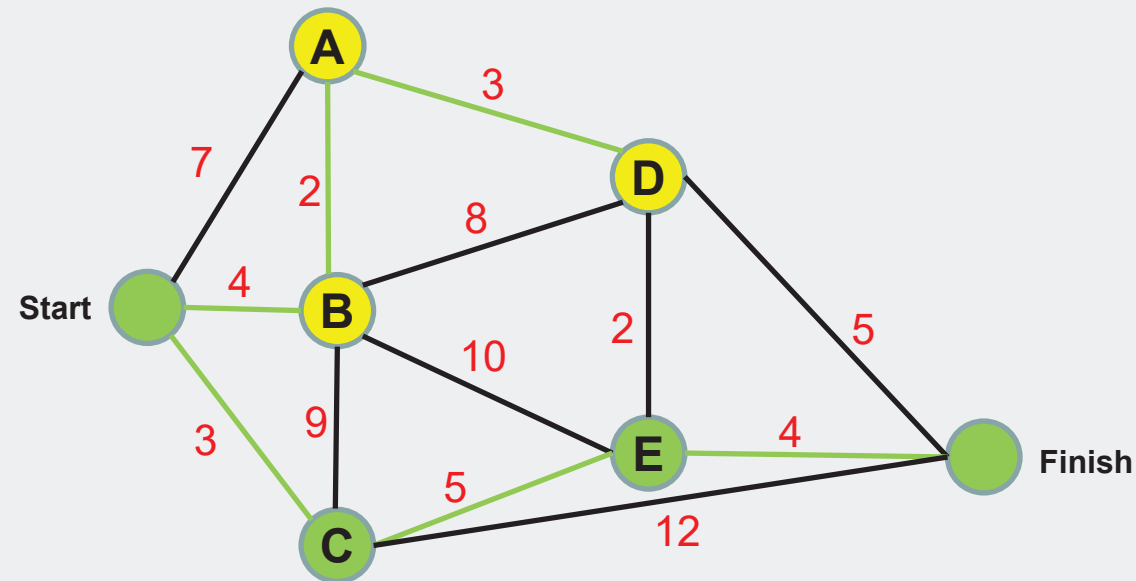
A	B	C	D	E	Finish
7, start	4, start	3, start			
7, start	4, start	*		8, C	15, C
6, B	*	*	12, B	8, C	15, C
*	*	*	9, A	8, C	15, C
*	*	*	9, A	*	12, E
*	*	*		*	



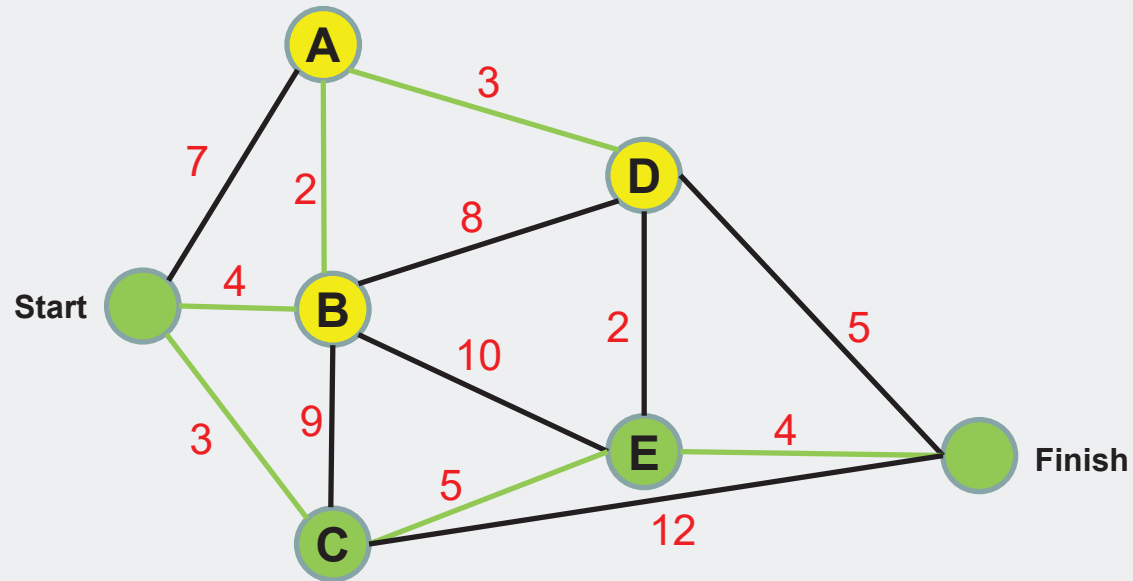
A	B	C	D	E	Finish
7, start	4, start	3, start			
7, start	4, start	*		8, C	15, C
6, B	*	*	12, B	8, C	15, C
*	*	*	9, A	8, C	15, C
*	*	*	9, A	*	12, E
*	*	*	*	*	12, E



A	B	C	D	E	Finish
7, start	4, start	3, start			
7, start	4, start	*		8, C	15, C
6, B	*	*	12, B	8, C	15, C
*	*	*	9, A	8, C	15, C
*	*	*	9, A	*	12, E
*	*	*	*	*	12, E



A	B	C	D	E	Finish
7, start	4, start	3, start			
7, start	4, start	*		8, C	15, C
6, B	*	*	12, B	8, C	15, C
*	*	*	9, A	8, C	15, C
*	*	*	9, A	*	12, E
*	*	*	*	*	12, E

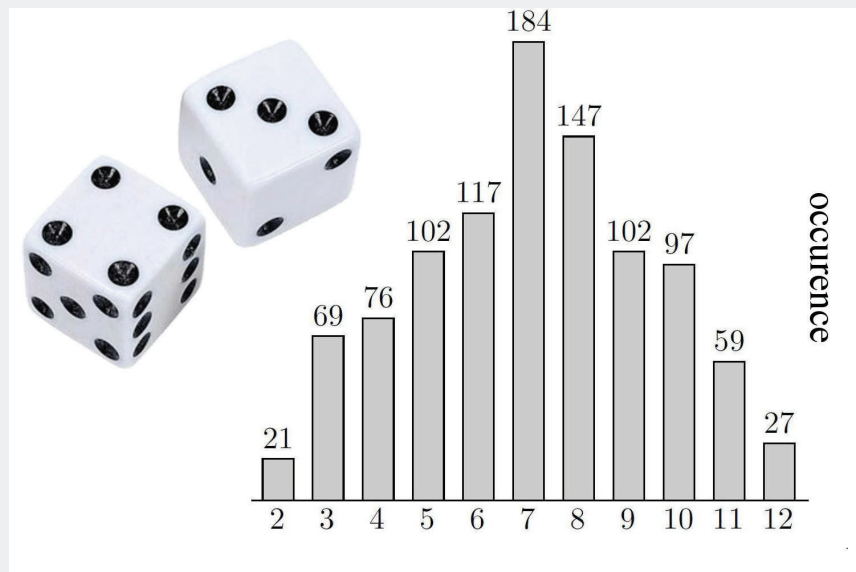


- The shortest route is given by: Start→C→E→Finish

Probability distributions

A *random variable* X is a variable whose possible values are outcomes of a random experiment.

We will also use the term *stochastic* as a synonym for random.



Continuous random variables

- A random variable X is called continuous when it can take continuously many values, for simplicity we can just say that its values are the real numbers.
- For a continuous random variable we can specify the probability it falls in a **range** of values using the *density function*.

Normal random variables

The normal random variable is a continuous random variable that has a symmetric density function. The plot of the density function has a bell-shaped form.

$$\mathbb{P}(\mathcal{N}(\mu, \sigma^2) \leq t) = \int_{x=0}^t \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx.$$

The density of a normally distributed r.v.

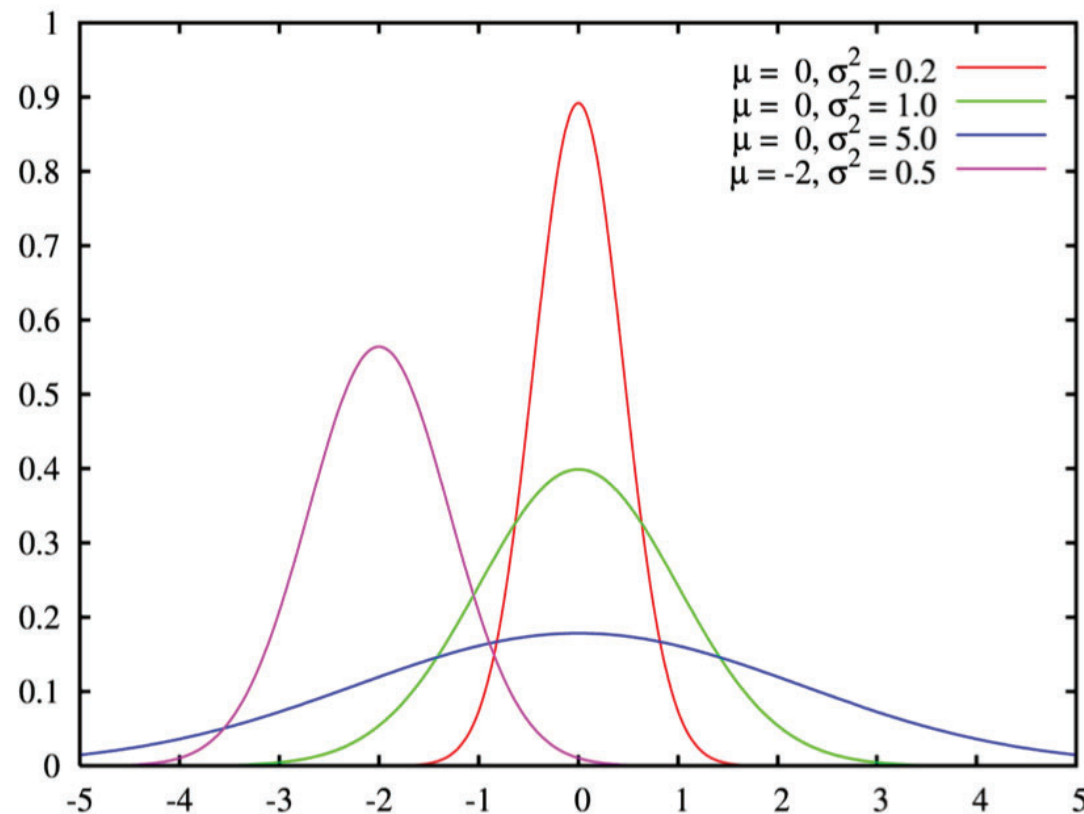


Figure 1.2.1. The density function of a normally distributed random variable with parameters μ and σ^2 .



***Time to give the microphone to
Nikki!***