Evolution of population genetics

March 12, 2019

1 Wright Fisher model

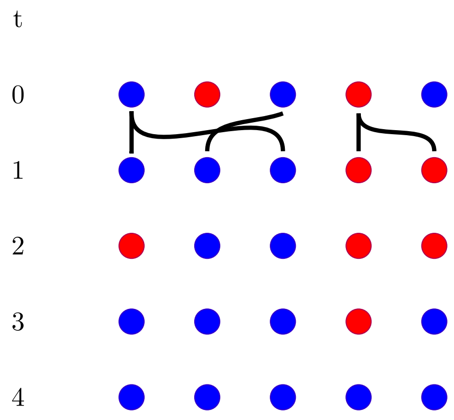
Recall the single colony Wright Fisher model of slide 1, see figure . Here each individual chooses uniformly at random an individual from the current population and adopt this type. This is the same as putting for all individuals at time 0 as a marble with their type (RED or BLUE) in an urn and draw a marble at random for each individual to determine their new type. While doing this we put the drawn marble each time back in the urn. Suppose we start with 2 red individuals and 3 blue individuals at time 0 and let X_n denote the number of red individuals in generation n.

A Show that the $\mathbb{P}(X_1 = 3) = \binom{5}{3}(\frac{2}{5})^3(\frac{3}{5})^2$ and $\mathbb{E}[X_1] = 2$.

B Suppose now that we start with N individuals and we know $X_0 = K$. Let $0 \leq l \leq N$, can you show that $\mathbb{P}(X_1 = l) = \binom{N}{l}(\frac{K}{N})^l(\frac{N-K}{N})^{N-l}$ and $\mathbb{E}[X_1] = K$

C Note that $\mathbb{E}[X_1] = X_0$, can you guess what $\mathbb{E}[X_n]$ will be? (Hint: suppose you know X_{n-1}, can you express the probability $\mathbb{P}(X_n = l)$ in terms of X_{n-1}? Try to compute $\mathbb{E}[X_n|X_{n-1}]$, this is the expectation of X_n if you know what X_{n-1} is.)

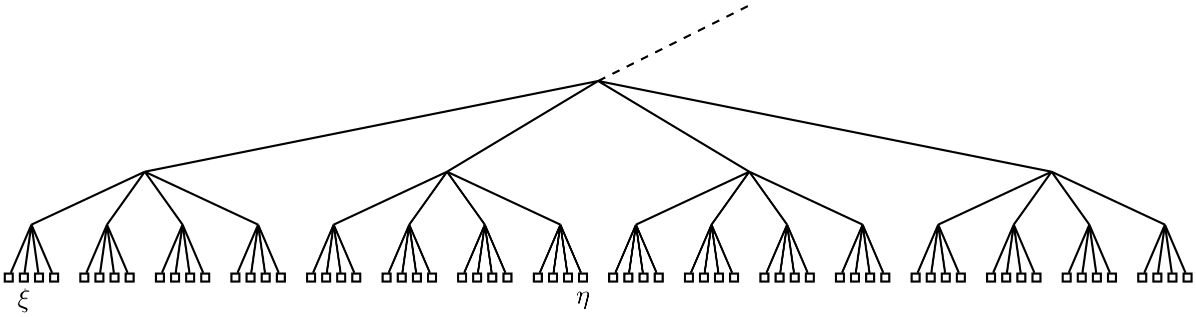
D Suppose there exists an generation m such that $X_m = 0$ or $X_m = N$. What do we know for X_n with $n > m$, so the generations after generation m?



wfmodel

Figure 1: Wright Fisher model with $N = 5$

hier



2 Hierarchical lattice Ω_4

Answer the question 1 and 2 by drawing it in figure 2.

1. Give the sequence which indicates the positions of ξ and η.
2. Draw a block of all individuals at distance 1 from ξ and at distance 2 and at distance 3.
3. What is the distance between ξ and η?

3 Loss of genetic diversity

In this question we compute what the probability is that two individuals are of the same type. We consider a Wright Fisher model with N individuals. The number of red individuals at time 0 is x_0, $1 \leq x_0 \leq N$.

A Show that two individuals a and b chosen at random from the population at time 1 have the same parent equals $\frac{1}{N}$. Show that the same holds for the probability that two individuals chosen at random from the population at time n have the same parent in generation $n-1$.

B From A it follows that the probability two individuals in generation n do not have the same parent equals $(1-\frac{1}{N})$. Show that the probability that two individuals in generation n have a the same ancestor in generation 0 equals $(1-\frac{1}{N})^n$

C We know that the initial number of red individuals $X_0 = x_0$. Show that the probability that the two individuals a and b in generation n are of a different type equals $(1-\frac{1}{N})^n \frac{x_0(N-x_0)}{N(N-1)}$.What is the probability that the two individuals a and b in generation n are of the same type?

D What happens with the probability computed in C if n gets larger?

E Suppose we know that all the N individuals in our population have the same ancestor. What do you think is the probability that this ancestor is RED?

F If we have a single colony do you think that one type always get extinct? Can you think of extensions of the model such that types get less often extinct?